9

Sequences and Series

TOPIC 1 Arithmetic Progression

1. The common difference of the A.P. $b_1, b_2, ..., b_m$ is 2 more than the common difference of A.P. $a_1, a_2, ..., a_n$. If $a_{40} = -159$, $a_{100} = -399$ and $b_{100} = a_{70}$, then b_1 is equal to: **[Sep. 06, 2020 (II)]**

(a) 81 (b) –127 (c) –81 (d) 127

2. If $3^{2\sin 2\alpha - 1}$, 14 and $3^{4-2\sin 2\alpha}$ are the first three terms of an A.P. for some α, then the sixth term of this A.P is: **[Sep. 05, 2020 (I)]**

(a) 66 (b) 81 (c) 65 (d) 78

3. If the sum of the first 20 terms of the series $\log_{(7^{1/2})} x + \log_{(7^{1/3})} x + \log_{(7^{1/4})} x + ...$ is 460, then x is equal to : **[Sep. 05, 2020 (II)]**

(a) 7^2 (b) $7^{1/2}$ (c) e^2 (d) $7^{46/21}$

4. Let $a_1, a_2, ..., a_n$ be a given A.P. whose common difference is an integer and $S_n = a_1 + a_2 + ... + a_n$. If $a_1 = 1$, $a_n = 300$ and $15 \le n \le 50$, then the ordered pair (S_{n-4}, a_{n-4}) is equal to : **[Sep. 04, 2020 (II)]**

(a) (2490, 249) (b) (2480, 249)
(c) (2480, 248) (d) (2490, 248)

5. If the first term of an A.P. is 3 and the sum of its first 25 terms is equal to the sum of its next 15 terms, then the common difference of this A.P. is : **[Sep. 03, 2020 (I)]**

(a) $\frac{1}{6}$ (b) $\frac{1}{5}$ (c) $\frac{1}{4}$ (d) $\frac{1}{7}$

6. In the sum of the series

$20 + 19\frac{3}{5} + 19\frac{1}{5} + 18\frac{4}{5} + ...$ upto n^{th} term is 488 and then n^{th} term is negative, then : **[Sep. 03, 2020 (II)]**

(a) $n = 60$ (b) n^{th} term is –4
(c) $n = 41$ (d) n^{th} term is $-4\frac{2}{5}$

7. If the sum of first 11 terms of an A.P., $a_1, a_2, a_3, ...$ is 0 $(a_1 \neq 0)$, then the sum of the A.P., $a_1, a_3, a_5, ..., a_{23}$ is ka_1, where k is equal to : **[Sep. 02, 2020 (II)]**

(a) $-\frac{121}{10}$ (b) $\frac{121}{10}$ (c) $\frac{72}{5}$ (d) $-\frac{72}{5}$

8. The number of terms common to the two A.P.'s 3, 7, 11, ..., 407 and 2, 9, 16, ..., 709 is ______. **[NA Jan. 9, 2020 (II)]**

9. If the 10^{th} term of an A.P. is $\frac{1}{20}$ and its 20^{th} term is $\frac{1}{10}$, then the sum of its first 200 terms is: **[Jan. 8, 2020 (II)]**

(a) 50 (b) $50\frac{1}{4}$ (c) 100 (d) $100\frac{1}{2}$

10. Let $f : R \to R$ be such that for all $x \in R$, $(2^{1+x} + 2^{1-x})$, $f(x)$ and $(3^x + 3^{-x})$ are in A.P., then the minimum value of $f(x)$ is: **[Jan. 8, 2020 (I)]**

(a) 2 (b) 3 (c) 0 (d) 4

11. Five numbers are in A.P., whose sum is 25 and product is 2520. If one of these five numbers is $-\frac{1}{2}$, then the greatest number amongst them is: **[Jan. 7, 2020 (I)]**

(a) 27 (b) 7 (c) $\frac{21}{2}$ (d) 16

12. Let S_n denote the sum of the first n terms of an A.P. If $S_4 = 16$ and $S_6 = -48$, then S_{10} is equal to : **[April 12, 2019 (I)]**

(a) –260 (b) –410 (c) –320 (d) –380

13. If $a_1, a_2, a_3, ...$ are in A.P. such that $a_1 + a_7 + a_{16} = 40$, then the sum of the first 15 terms of this A.P. is : **[April 12, 2019 (II)]**

(a) 200 (b) 280 (c) 120 (d) 150

14. If $a_1, a_2, a_3, ... a_n$ are in A.P. and $a_1 + a_4 + a_7 + ... + a_{16} = 114$, then $a_1 + a_6 + a_{11} + a_{16}$ is equal to : **[April 10, 2019 (I)]**

(a) 98 (b) 76 (c) 38 (d) 64

15. Let the sum of the first n terms of a non-constant A.P., $a_1, a_2, a_3, \ldots\ldots\ldots\ldots$ be $50n + \frac{n(n-7)}{2}A$, where A is a constant. If d is the common difference of this A.P., then the ordered pair (d, a_{50}) is equal to: **[April 09, 2019 (I)]**

(a) $(50, 50+46A)$ (b) $(50, 50+45A)$
(c) $(A, 50+45A)$ (d) $(A, 50+46A)$

16. Let $\sum_{k=1}^{10} f(a+k) = 16(2^{10}-1)$, where the function f satisfies $f(x+y) = f(x)f(y)$ for all natural numbers x, y and $f(a) = 2$. Then the natural number 'a' is: **[April 09, 2019 (I)]**

(a) 2 (b) 16 (c) 4 (d) 3

17. If the sum and product of the first three terms in an A.P. are 33 and 1155, respectively, then a value of its 11th term is: **[April 09, 2019 (II)]**

(a) –35 (b) 25 (c) –36 (d) –25

18. The sum of all natural numbers 'n' such that $100 < n < 200$ and H.C.F. $(91, n) > 1$ is : **[April 08, 2019 (I)]**

(a) 3203 (b) 3303 (c) 3221 (d) 3121

19. If ${}^nC_4$, ${}^nC_5$ and ${}^nC_6$ are in A.P., then n can be : **[Jan. 12, 2019 (II)]**

(a) 9 (b) 14 (c) 11 (d) 12

20. If 19th term of a non-zero A.P. is zero, then its (49th term) : (29th term) is : **[Jan. 11, 2019 (II)]**

(a) 4 : 1 (b) 1 : 3 (c) 3 : 1 (d) 2 : 1

21. The sum of all two digit positive numbers which when divided by 7 yield 2 or 5 as remainder is: **[Jan. 10, 2019 (I)]**

(a) 1256 (b) 1465 (c) 1365 (d) 1356

22. Let $a_1, a_2, \ldots\ldots, a_{30}$ be an A.P., $S = \sum_{i=1}^{30} a_i$ and $T = \sum_{i=1}^{15} a_{(2i-1)}$.

If $a_5 = 27$ and $S - 2T = 75$, then a_{10} is equal to: **[Jan. 09, 2019 (I)]**

(a) 52 (b) 57 (c) 47 (d) 42

23. Let $\frac{1}{x_1}, \frac{1}{x_2}, \frac{1}{x_3}, \ldots\ldots, (x_i \neq 0$ for $i = 1, 2, \ldots, n)$ be in A.P. such that $x_1 = 4$ and $x_{21} = 20$. If n is the least positive integer for which $x_n > 50$, then $\sum_{i=1}^{n}\left(\frac{1}{x_i}\right)$ is equal to. **[Online April 16, 2018]**

(a) 3 (b) $\frac{13}{8}$ (c) $\frac{13}{4}$ (d) $\frac{1}{8}$

24. If $x_1, x_2, \ldots\ldots, x_n$ and $\frac{1}{h_1}, \frac{1}{h^2}, \ldots\ldots \frac{1}{h_n}$ are two A.P's such that $x_3 = h_2 = 8$ and $x_8 = h_7 = 20$, then $x_5 . h_{10}$ equals. **[Online April 15, 2018]**

(a) 2560 (b) 2650 (c) 3200 (d) 1600

25. Let $a_1, a_2, a_3, \ldots, a_{49}$ be in A.P. such that $\sum_{k=0}^{12} a_{4k+1} = 416$ and $a_9 + a_{43} = 66$. If $a_1^2 + a_2^2 + \ldots + a_{17}^2 = 140m$, then m is equal to : **[2018]**

(a) 68 (b) 34 (c) 33 (d) 66

26. For any three positive real numbers a, b and c, $9(25a^2 + b^2) + 25(c^2 - 3ac) = 15b(3a + c)$. Then : **[2017]**

(a) a, b and c are in G.P.
(b) b, c and a are in G.P.
(c) b, c and a are in A.P.
(d) a, b and c are in A.P.

27. If three positive numbers a, b and c are in A.P. such that $abc = 8$, then the minimum possible value of b is : **[Online April 9, 2017]**

(a) 2 (b) $4^{\frac{1}{3}}$ (c) $4^{\frac{2}{3}}$ (d) 4

28. Let $a_1, a_2, a_3, \ldots, a_n$, be in A.P. If $a_3 + a_7 + a_{11} + a_{15} = 72$, then the sum of its first 17 terms is equal to : **[Online April 10, 2016]**

(a) 306 (b) 204 (c) 153 (d) 612

29. Let α and β be the roots of equation $px^2 + qx + r = 0, p \neq 0$. If p, q, r are in A.P and $\frac{1}{\alpha} + \frac{1}{\beta} = 4$, then the value of $|\alpha - \beta|$ is: **[2014]**

(a) $\frac{\sqrt{34}}{9}$ (b) $\frac{2\sqrt{13}}{9}$ (c) $\frac{\sqrt{61}}{9}$ (d) $\frac{2\sqrt{17}}{9}$

30. The sum of the first 20 terms common between the series 3 + 7 + 11 + 15 + ... and 1 + 6 + 11 + 16 + ..., is **[Online April 11, 2014]**

(a) 4000 (b) 4020 (c) 4200 (d) 4220

31. Given an A.P. whose terms are all positive integers. The sum of its first nine terms is greater than 200 and less than 220. If the second term in it is 12, then its 4th term is: **[Online April 9, 2014]**

(a) 8 (b) 16 (c) 20 (d) 24

32. If $a_1, a_2, a_3, \ldots, a_n, \ldots$ are in A.P. such that $a_4 - a_7 + a_{10} = m$, then the sum of first 13 terms of this A.P., is : **[Online April 23, 2013]**

(a) 10m (b) 12m (c) 13m (d) 15m

33. Given sum of the first n terms of an A.P. is $2n + 3n^2$. Another A.P. is formed with the same first term and double of the common difference, the sum of n terms of the new A.P. is : **[Online April 22, 2013]**

(a) $n + 4n^2$ (b) $6n^2 - n$ (c) $n^2 + 4n$ (d) $3n + 2n^2$

34. Let $a_1, a_2, a_3,...$ be an A.P, such that

$$\frac{a_1 + a_2 + ... + a_p}{a_1 + a_2 + a_3 + ... + a_q} = \frac{p^3}{q^3}; p \neq q. \text{ Then } \frac{a_6}{a_{21}} \text{ is equal to:}$$

[Online April 9, 2013]

(a) $\frac{41}{11}$ (b) $\frac{31}{121}$ (c) $\frac{11}{41}$ (d) $\frac{121}{1861}$

35. If 100 times the 100^{th} term of an AP with non zero common difference equals the 50 times its 50^{th} term, then the 150^{th} term of this AP is : **[2012]**

(a) -150 (b) 150 times its 50^{th} term
(c) 150 (d) Zero

36. If the A.M. between p^{th} and q^{th} terms of an A.P. is equal to the A.M. between r^{th} and s^{th} terms of the same A.P., then $p + q$ is equal to **[Online May 26, 2012]**

(a) $r + s - 1$ (b) $r + s - 2$ (c) $r + s + 1$ (d) $r + s$

37. Suppose θ and $\phi (\neq 0)$ are such that $\sec(\theta + \phi)$, $\sec\theta$ and $\sec(\theta - \phi)$ are in A.P. If $\cos\theta = k\cos\left(\frac{\phi}{2}\right)$ for some k, then k is equal to **[Online May 19, 2012]**

(a) $\pm\sqrt{2}$ (b) ± 1 (c) $\pm\frac{1}{\sqrt{2}}$ (d) ± 2

38. Let a_n be the n^{th} term of an A.P. If $\sum_{r=1}^{100} a_{2r} = \alpha$ and $\sum_{r=1}^{100} a_{2r-1} = \beta$, then the common difference of the A.P. is **[2011]**

(a) $\alpha - \beta$ (b) $\frac{\alpha - \beta}{100}$ (c) $\beta - \alpha$ (d) $\frac{\alpha - \beta}{200}$

39. A man saves ₹ 200 in each of the first three months of his service. In each of the subsequent months his saving increases by ₹ 40 more than the saving of immediately previous month. His total saving from the start of service will be ₹ 11040 after **[2011]**

(a) 19 months (b) 20 months
(c) 21 months (d) 18 months

40. A person is to count 4500 currency notes. Let a_n denote the number of notes he counts in the n^{th} minute. If $a_1 = a_2 = ... = a_{10} = 150$ and $a_{10}, a_{11}, ...$ are in an AP with common difference -2, then the time taken by him to count all notes is **[2010]**

(a) 34 minutes (b) 125 minutes
(c) 135 minutes (d) 24 minutes

41. Let $a_1, a_2, a_3 ....$ be terms on A.P. If

$$\frac{a_1 + a_2 + ...a_p}{a_1 + a_2 + ... + a_q} = \frac{p^2}{q^2}, \; p \neq q, \text{ then } \frac{a_6}{a_{21}} \text{ equals}$$

[2006]

(a) $\frac{41}{11}$ (b) $\frac{7}{2}$ (c) $\frac{2}{7}$ (d) $\frac{11}{41}$

42. If the coefficients of rth, $(r + 1)$th, and $(r + 2)$th terms in the the binomial expansion of $(1 + y)^m$ are in A.P., then m and r satisfy the equation **[2005]**

(a) $m^2 - m(4r - 1) + 4r^2 - 2 = 0$
(b) $m^2 - m(4r + 1) + 4r^2 + 2 = 0$
(c) $m^2 - m(4r + 1) + 4r^2 - 2 = 0$
(d) $m^2 - m(4r - 1) + 4r^2 + 2 = 0$

43. Let T_r be the rth term of an A.P. whose first term is a and common difference is d. If for some positive integers $m, n, m \neq n, T_m = \frac{1}{n}$ and $T_n = \frac{1}{m}$, then $a - d$ equals **[2004]**

(a) $\frac{1}{m} + \frac{1}{n}$ (b) 1 (c) $\frac{1}{mn}$ (d) 0

44. If $1, \log_9(3^{1-x} + 2), \log_3(4.3^x - 1)$ are in A.P. then x equals **[2002]**

(a) $\log_3 4$ (b) $1 - \log_3 4$
(c) $1 - \log_4 3$ (d) $\log_4 3$

TOPIC 2 Geometric Progression

45. If $f(x + y) = f(x)f(y)$ and $\sum_{x=1}^{\infty} f(x) = 2$, $x, y \in N$, where N is the set of all natural numbers, then the value of $\frac{f(4)}{f(2)}$ is : **[Sep. 06, 2020 (I)]**

(a) $\frac{2}{3}$ (b) $\frac{1}{9}$ (c) $\frac{1}{3}$ (d) $\frac{4}{9}$

46. Let a, b, c, d and p be any non zero distinct real numbers such that $(a^2 + b^2 + c^2)p^2 - 2(ab + bc + cd)p + (b^2 + c^2 + d^2) = 0$. Then : **[Sep. 06, 2020 (I)]**

(a) a, c, p are in A.P. (b) a, c, p are in G.P.
(c) a, b, c, d are in G.P. (d) a, b, c, d are in A.P.

47. Suppose that a function $f: R \to R$ satisfies $f(x+y) = f(x)f(y)$ for all $x, y \in R$ and $f(a) = 3$. If $\sum_{i=1}^{n} f(i) = 363$, then n is equal to ______. **[NA Sep. 06, 2020 (II)]**

48. If $2^{10} + 2^9 \cdot 3^1 + 2^8 \cdot 3^2 + \ldots + 2 \times 3^9 + 3^{10} = S - 2^{11}$ then S is equal to: **[Sep. 05, 2020 (I)]**

(a) $3^{11} - 2^{12}$ (b) 3^{11}

(c) $\frac{3^{11}}{2} + 2^{10}$ (d) $2 \cdot 3^{11}$

49. If the sum of the second, third and fourth terms of a positive term G.P. is 3 and the sum of its sixth, seventh and eighth terms is 243, then the sum of the first 50 terms of this G.P. is: **[Sep. 05, 2020 (II)]**

(a) $\frac{1}{26}(3^{49} - 1)$ (b) $\frac{1}{26}(3^{50} - 1)$

(c) $\frac{2}{13}(3^{50} - 1)$ (d) $\frac{1}{13}(3^{50} - 1)$

50. Let α and β be the roots of $x^2 - 3x + p = 0$ and γ and δ be the roots of $x^2 - 6x + q = 0$. If $\alpha, \beta, \gamma, \delta$ form a geometric progression. Then ratio $(2q + p) : (2q - p)$ is : **[Sep. 04, 2020 (I)]**

(a) 3 : 1 (b) 9 : 7 (c) 5 : 3 (d) 33 : 31

51. The value of $(0.16)\log_{2.5}\left(\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \ldots \text{ to } \infty\right)$ is equal to _________ . **[NA Sep. 03, 2020 (I)]**

52. The sum of the first three terms of a G.P. is S and their product is 27. Then all such S lie in : **[Sep. 02, 2020 (I)]**

(a) $(-\infty, -9] \cup [3, \infty)$ (b) $[-3, \infty)$

(c) $(-\infty, -3] \cup [9, \infty)$ (d) $(-\infty, 9]$

53. If $|x| < 1, |y| < 1$ and $x \neq y$, then the sum to infinity of the following series **[Sep. 02, 2020 (I)]**

$(x + y) + (x^2 + xy + y^2) + (x^3 + x^2y + xy^2 + y^3) + \ldots$ is :

(a) $\frac{x + y - xy}{(1+x)(1+y)}$ (b) $\frac{x + y + xy}{(1+x)(1+y)}$

(c) $\frac{x + y - xy}{(1-x)(1-y)}$ (d) $\frac{x + y + xy}{(1-x)(1-y)}$

54. Let S be the sum of the first 9 terms of the series :

$\{x + ka\} + \{x^2 + (k+2)a\} + \{x^3 + (k+4)a\}$
$+ \{x^4 + (k+6)a\} + \ldots$ where $a \neq 0$ and $x \neq 1$. If

$S = \frac{x^{10} - x + 45a(x-1)}{x-1}$, then k is equal to : **[Sep. 02, 2020 (II)]**

(a) -5 (b) 1 (c) -3 (d) 3

55. The product $2^{\frac{1}{4}} \cdot 4^{\frac{1}{16}} \cdot 8^{\frac{1}{48}} \cdot 16^{\frac{1}{128}} \ldots$ to ∞ is equal to: **[Jan. 9, 2020 (I)]**

(a) $2^{\frac{1}{2}}$ (b) $2^{\frac{1}{4}}$ (c) 1 (d) 2

56. Let a_n be the n^{th} term of a G.P. of positive terms.

If $\sum_{n=1}^{100} a_{2n+1} = 200$ and $\sum_{n=1}^{100} a_{2n} = 100$, then $\sum_{n=1}^{200} a_n$ is equal to : **[Jan. 9, 2020 (II)]**

(a) 300 (b) 225 (c) 175 (d) 150

57. If $x = \sum_{n=0}^{\infty} (-1)^n \tan^{2n}\theta$ and $y = \sum_{n=0}^{\infty} \cos^{2n}\theta$, for $0 < \theta < \frac{\pi}{4}$, then : **[Jan. 9, 2020 (II)]**

(a) $x(1 + y) = 1$ (b) $y(1 - x) = 1$

(c) $y(1 + x) = 1$ (d) $x(1 - y) = 1$

58. The greatest positive integer k, for which $49^k + 1$ is a factor of the sum $49^{125} + 49^{124} + \ldots + 49^2 + 49 + 1$, is: **[Jan. 7, 2020 (I)]**

(a) 32 (b) 63 (c) 60 (d) 65

59. Let $a_1, a_2, a_3, \ldots$ be a G. P. such that $a_1 < 0$, $a_1 + a_2 = 4$ and $a_3 + a_4 = 16$. If $\sum_{i=1}^{9} a_i = 4\lambda$, then λ is equal to: **[Jan. 7, 2020 (II)]**

(a) –513 (b) –171 (c) 171 (d) $\frac{511}{3}$

60. The coefficient of x^7 in the expression $(1 + x)^{10} + x(1 + x)^9 + x^2(1 + x)^8 + \ldots + x^{10}$ is: **[Jan. 7, 2020 (II)]**

(a) 210 (b) 330 (c) 120 (d) 420

61. If α, β and γ are three consecutive terms of a non-constant G.P. such that the equations $\alpha x^2 + 2\beta x + \gamma = 0$ and $x^2 + x - 1 = 0$ have a common root, then $\alpha(\beta + \gamma)$ is equal to : **[April 12, 2019 (II)]**

(a) 0 (b) $\alpha\beta$ (c) $\alpha\gamma$ (d) $\beta\gamma$

62. Let a, b and c be in G.P. with common ratio r, where $a \neq 0$ and $0 < r \leq \frac{1}{2}$. If $3a$, $7b$ and $15c$ are the first three terms of an A.P., then the 4^{th} term of this A.P. is : **[April 10, 2019 (II)]**

(a) $\frac{2}{3}a$ (b) $5a$ (c) $\frac{7}{3}a$ (d) a

63. If three distinct numbers a, b, c are in G.P. and the equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ have a common root, then which one of the following statements is correct? **[April 08, 2019 (II)]**

(a) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P. (b) d, e, f are in A.P.

(c) d, e, f are in G.P. (d) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in G.P.

64. The product of three consecutive terms of a G.P. is 512. If 4 is added to each of the first and the second of these terms, the three terms now form an A.P. Then the sum of the original three terms of the given G.P. is : **[Jan. 12, 2019 (I)]**

(a) 36 (b) 32 (c) 24 (d) 28

65. Let α and β be the roots of the quadratic equation $x^2 \sin\theta - x(\sin\theta\cos\theta + 1) + \cos\theta = 0$ $(0 < \theta < 45°)$, and $\alpha < \beta$. Then $\sum_{n=0}^{\infty}\left(\alpha^n + \frac{(-1)^n}{\beta^n}\right)$ is equal to : **[Jan. 11, 2019 (II)]**

(a) $\frac{1}{1-\cos\theta} - \frac{1}{1+\sin\theta}$ (b) $\frac{1}{1+\cos\theta} + \frac{1}{1-\sin\theta}$

(c) $\frac{1}{1-\cos\theta} + \frac{1}{1+\sin\theta}$ (d) $\frac{1}{1+\cos\theta} - \frac{1}{1-\sin\theta}$

66. Let $a_1, a_2, ..., a_{10}$ be a G.P. If $\frac{a_3}{a_1} = 25$, then $\frac{a_9}{a_5}$ equals : **[Jan. 11, 2019 (I)]**

(a) 5^4 (b) $4(5^2)$ (c) 5^3 (d) $2(5^2)$

67. The sum of an infinite geometric series with positive terms is 3 and the sum of the cubes of its terms is $\frac{27}{19}$. Then the common ratio of this series is : **[Jan. 11, 2019 (I)]**

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{2}{9}$ (d) $\frac{4}{9}$

68. Let $S_n = 1 + q + q^2 + ... + q^n$ and

$$T_n = 1 + \left(\frac{q+1}{2}\right) + \left(\frac{q+1}{2}\right)^2 + ... + \left(\frac{q+1}{2}\right)^n$$

where q is a real number and $q \neq 1$. If $^{101}C_1 + {}^{101}C_2 \cdot S_1 + ... + {}^{101}C_{101} \cdot S_{100} = \alpha T_{100}$, then α is equal to : **[Jan. 11, 2019 (II)]**

(a) 2^{99} (b) 202 (c) 200 (d) 2^{100}

69. Let a, b and c be the 7^{th}, 11^{th} and 13^{th} terms respectively of a non-constant A.P. If these are also the three consecutive terms of a G.P., then $\frac{a}{c}$ is equal to: **[Jan. 09, 2019 (II)]**

(a) 2 (b) $\frac{1}{2}$ (c) $\frac{7}{13}$ (d) 4

70. If a, b and c be three distinct real numbers in G.P. and $a + b + c = xb$, then x cannot be: **[Jan. 09, 2019 (I)]**

(a) -2 (b) -3 (c) 4 (d) 2

71. If b is the first term of an infinite G. P whose sum is five, then b lies in the interval. **[Online April 15, 2018]**

(a) $(-\infty, -10)$ (b) $(10, \infty)$

(c) $(0, 10)$ (d) $(-10, 0)$

72. Let $A_n = \left(\frac{3}{4}\right) - \left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^3 - ... + (-1)^{n-1}\left(\frac{3}{4}\right)^n$ and $B_n = 1 - A_n$. Then, the least odd natural number p, so that $B_n > A_n$, for all $n \geq p$ is **[Online April 15, 2018]**

(a) 5 (b) 7 (c) 11 (d) 9

73. If a, b, c are in A.P. and a^2, b^2, c^2 are in G.P. such that $a < b < c$ and $a + b + c = \frac{3}{4}$, then the value of a is **[Online April 15, 2018]**

(a) $\frac{1}{4} - \frac{1}{3\sqrt{2}}$ (b) $\frac{1}{4} - \frac{1}{4\sqrt{2}}$

(c) $\frac{1}{4} - \frac{1}{\sqrt{2}}$ (d) $\frac{1}{4} - \frac{1}{2\sqrt{2}}$

74. If the 2^{nd}, 5^{th} and 9^{th} terms of a non-constant A.P. are in G.P., then the common ratio of this G.P. is : **[2016]**

(a) 1 (b) $\frac{7}{4}$ (c) $\frac{8}{5}$ (d) $\frac{4}{3}$

75. Let $z = 1 + ai$ be a complex number, $a > 0$, such that z^3 is a real number. Then the sum $1 + z + z^2 + ... + z^{11}$ is equal to: **(Online April 10, 2016)**

(a) $1365\sqrt{3}i$ (b) $-1365\sqrt{3}i$

(c) $-1250\sqrt{3}i$ (d) $1250\sqrt{3}i$

76. If m is the A.M. of two distinct real numbers l and n $(l, n > 1)$ and G_1, G_2 and G_3 are three geometric means between l and n, then $G_1^4 + 2G_2^4 + G_3^4$ equals. **[2015]**

(a) $4\,lmn^2$ (b) $4\,l^2m^2n^2$ (c) $4\,l^2mn$ (d) $4\,lm^2n$

77. The sum of the 3^{rd} and the 4^{th} terms of a G.P. is 60 and the product of its first three terms is 1000. If the first term of this G.P. is positive, then its 7^{th} term is : **[Online April 11, 2015]**

(a) 7290 (b) 640 (c) 2430 (d) 320

78. Three positive numbers form an increasing G. P. If the middle term in this G.P. is doubled, the new numbers are in A.P. then the common ratio of the G.P. is: **[2014]**

(a) $2-\sqrt{3}$ (b) $2+\sqrt{3}$

(c) $\sqrt{2}+\sqrt{3}$ (d) $3+\sqrt{2}$

79. The least positive integer n such that

$1-\frac{2}{3}-\frac{2}{3^2}-....-\frac{2}{3^{n-1}}<\frac{1}{100}$, is: **[Online April 12, 2014]**

(a) 4 (b) 5 (c) 6 (d) 7

80. In a geometric progression, if the ratio of the sum of first 5 terms to the sum of their reciprocals is 49, and the sum of the first and the third term is 35. Then the first term of this geometric progression is: **[Online April 11, 2014]**

(a) 7 (b) 21 (c) 28 (d) 42

81. The coefficient of x^{50} in the binomial expansion of $(1+x)^{1000}+x(1+x)^{999}+x^2(1+x)^{998}+....+x^{1000}$ is: **[Online April 11, 2014]**

(a) $\frac{(1000)!}{(50)!(950)!}$ (b) $\frac{(1000)!}{(49)!(951)!}$

(c) $\frac{(1001)!}{(51)!(950)!}$ (d) $\frac{(1001)!}{(50)!(951)!}$

82. Given a sequence of 4 numbers, first three of which are in G.P. and the last three are in A.P. with common difference six. If first and last terms of this sequence are equal, then the last term is : **[Online April 25, 2013]**

(a) 16 (b) 8 (c) 4 (d) 2

83. If a, b, c, d and p are distinct real numbers such that $(a^2+b^2+c^2)p^2-2p(ab+bc+cd)+(b^2+c^2+d^2)\le 0$, then **[Online May 12, 2012]**

(a) a, b, c, d are in A.P. (b) $ab = cd$

(c) $ac = bd$ (d) a, b, c, d are in G.P.

84. The difference between the fourth term and the first term of a Geometrical Progresssion is 52. If the sum of its first three terms is 26, then the sum of the first six terms of the progression is **[Online May 7, 2012]**

(a) 63 (b) 189 (c) 728 (d) 364

85. The first two terms of a geometric progression add up to 12. the sum of the third and the fourth terms is 48. If the terms of the geometric progression are alternately positive and negative, then the first term is **[2008]**

(a) –4 (b) –12 (c) 12 (d) 4

86. In a geometric progression consisting of positive terms, each term equals the sum of the next two terms. Then the common ratio of its progression is equals **[2007]**

(a) $\sqrt{5}$ (b) $\frac{1}{2}(\sqrt{5}-1)$

(c) $\frac{1}{2}(1-\sqrt{5})$ (d) $\frac{1}{2}\sqrt{5}$

87. The value of $\sum_{k=1}^{10}\left(\sin\frac{2k\pi}{11}+i\cos\frac{2k\pi}{11}\right)$ is **[2006]**

(a) i (b) 1 (c) -1 (d) $-i$

88. If the expansion in powers of x of the function $\frac{1}{(1-ax)(1-bx)}$ is $a_0+a_1x+a_2x^2+a_3x^3......$ then a_n is **[2006]**

(a) $\frac{b^n-a^n}{b-a}$ (b) $\frac{a^n-b^n}{b-a}$

(c) $\frac{a^{n+1}-b^{n+1}}{b-a}$ (d) $\frac{b^{n+1}-a^{n+1}}{b-a}$

89. Let two numbers have arithmetic mean 9 and geometric mean 4. Then these numbers are the roots of the quadratic equation **[2004]**

(a) $x^2-18x-16=0$ (b) $x^2-18x+16=0$

(c) $x^2+18x-16=0$ (d) $x^2+18x+16=0$

90. Sum of infinite number of terms of GP is 20 and sum of their square is 100. The common ratio of GP is **[2002]**

(a) 5 (b) 3/5 (c) 8/5 (d) 1/5

91. Fifth term of a GP is 2, then the product of its 9 terms is **[2002]**

(a) 256 (b) 512

(c) 1024 (d) none of these

92. The sum of all values of $\theta\in\left(0,\frac{\pi}{2}\right)$ satisfying

$\sin^2 2\theta+\cos^4 2\theta=\frac{3}{4}$ is: **[Jan. 10, 2019 (I)]**

(a) π (b) $\frac{5\pi}{4}$ (c) $\frac{\pi}{2}$ (d) $\frac{3\pi}{8}$

TOPIC 3 Harmonic Progression, Relation Between A. M., G. M. and H.M. of two Positive Numbers

93. If m arithmetic means (A.Ms) and three geometric means (G.Ms) are inserted between 3 and 243 such that 4^{th} A.M. is equal to 2^{nd} G.M., then m is equal to ____________. **[Sep. 03, 2020 (II)]**

94. If the arithmetic mean of two numbers a and b, a > b > 0, is five times their geometric mean, then $\frac{a+b}{a-b}$ is equal to : **[Online April 8, 2017]**

(a) $\frac{\sqrt{6}}{2}$ (b) $\frac{3\sqrt{2}}{4}$ (c) $\frac{7\sqrt{3}}{12}$ (d) $\frac{5\sqrt{6}}{12}$

95. If A > 0, B > 0 and $A+B=\frac{\pi}{6}$, then the minimum value of tanA + tanB is : **[Online April 10, 2016]**

(a) $\sqrt{3}-\sqrt{2}$ (b) $4-2\sqrt{3}$

(c) $\frac{2}{\sqrt{3}}$ (d) $2-\sqrt{3}$

96. Let x, y, z be positive real numbers such that x + y + z = 12 and $x^3y^4z^5 = (0.1)(600)^3$. Then $x^3 + y^3 + z^3$ is equal to : **[Online April 9, 2016]**

(a) 342 (b) 216 (c) 258 (d) 270

97. Let G be the geometric mean of two positive numbers a and b, and M be the arithmetic mean of $\frac{1}{a}$ and $\frac{1}{b}$. If $\frac{1}{M}$: G is 4 : 5, then a : b can be: **[Online April 12, 2014]**

(a) 1 : 4 (b) 1 : 2 (c) 2 : 3 (d) 3 : 4

98. If $a_1, a_2, ..., a_n$ are in H.P., then the expression $a_1a_2 + a_2a_3 + ... + a_{n-1}a_n$ is equal to **[2006]**

(a) $n(a_1 - a_n)$ (b) $(n-1)(a_1 - a_n)$

(c) na_1a_n (d) $(n-1)a_1a_n$

99. If $x=\sum_{n=0}^{\infty} a^n$, $y=\sum_{n=0}^{\infty} b^n$, $z=\sum_{n=0}^{\infty} c^n$ where a, b, c are in A.P and $|a|<1, |b|<1, |c|<1$ then x, y, z are in **[2005]**

(a) G.P.
(b) A.P.
(c) Arithmetic - Geometric Progression
(d) H.P.

100. If the sum of the roots of the quadratic equation $ax^2 + bx + c = 0$ is equal to the sum of the squares of their reciprocals, then $\frac{a}{c}, \frac{b}{a}$ and $\frac{c}{b}$ are in **[2003]**

(a) Arithmetic - Geometric Progression
(b) Arithmetic Progression
(c) Geometric Progression
(d) Harmonic Progression.

TOPIC 4 Arithmetic-Geometric Sequence (A.G.S.), Some Special Sequences

101. If $1+(1-2^2\cdot 1)+(1-4^2\cdot 3)+(1-6^2\cdot 5)+.......+(1-20^2\cdot 19)=\alpha-220\beta$, then an ordered pair (α, β) is equal to : **[Sep. 04, 2020 (I)]**

(a) (10, 97) (b) (11, 103)
(c) (10, 103) (d) (11, 97)

102. Let $f : \mathbf{R} \to \mathbf{R}$ be a function which satisfies $f(x+y) = f(x) + f(y), \forall x, y \in \mathbf{R}$. If f (a) = 2 and $g(n)=\sum_{k=1}^{(n-1)} f(k), n \in \mathbf{N}$, then the value of n, for which $g(n) = 20$, is : **[Sep. 02, 2020 (II)]**

(a) 5 (b) 20 (c) 4 (d) 9

103. The sum, $\sum_{n=1}^{7} \frac{n(n+1)(2n+1)}{4}$ is equal to __________. **[Jan. 8, 2020 (II)]**

104. The sum $\sum_{k=1}^{20}(1+2+3+...+k)$ is _______. **[Jan. 8, 2020 (I)]**

105. If the sum of the first 40 terms of the series, 3 + 4 + 8 + 9 + 13 + 14 + 18 + 19 + ... is (102)m, then m is equal to: **[Jan. 7, 2020 (II)]**

(a) 20 (b) 25 (c) 5 (d) 10

106. For $x \in R$, let [x] denote the greatest integer $\le x$, then the sum of the series

$$\left[-\frac{1}{3}\right]+\left[-\frac{1}{3}-\frac{1}{100}\right]+\left[-\frac{1}{3}-\frac{2}{100}\right]+\cdots+\left[-\frac{1}{3}-\frac{99}{100}\right] \text{ is}$$

[April 12, 2019 (I)]

(a) –153 (b) –133 (c) –131 (d) –135

107. The sum $\frac{3\times 1^3}{1^2}+\frac{5\times(1^3+2^3)}{1^2+2^2}+\frac{7\times(1^3+2^3+3^3)}{1^2+2^2+3^2})+......$ upto 10^{th} term, is : **[April 10, 2019 (I)]**

(a) 680 (b) 600 (c) 660 (d) 620

108. The sum $1+\frac{1^3+2^3}{1+2}+\frac{1^3+2^3+3^3}{1+2+3}+......+\frac{1^3+2^3+3^3+...+15^3}{1+2+3+...+15}-\frac{1}{2}(1+2+3+...+15$ is equal to : **[April 10, 2019 (II)]**

(a) 620 (b) 1240 (c) 1860 (d) 660

109. The sum of the series $1 + 2 \times 3 + 3 \times 5 + 4 \times 7 + \ldots$ upto 11^{th} term is: **[April 09, 2019 (II)]**

(a) 915 (b) 946 (c) 945 (d) 916

110. Some identical balls are arranged in rows to form an equilateral triangle. The first row consists of one ball, the second row consists of two balls and so on. If 99 more identical balls are added to the total number of balls used in forming the equilateral triangle, then all these balls can be arranged in a square whose each side contains exactly 2 balls less than the number of balls each side of the triangle contains. Then the number of balls used to form the equilateral triangle is: **[April 09, 2019 (II)]**

(a) 157 2) 262 (c) 225 (d) 190

111. The sum $\sum_{k=1}^{20} k\frac{1}{2^k}$ is equal to : **[April 08, 2019 (II)]**

(a) $2-\frac{3}{2^{17}}$ (b) $1-\frac{11}{2^{20}}$ (c) $2-\frac{11}{2^{19}}$ (d) $2-\frac{21}{2^{20}}$

112. Let $S_k = \frac{1+2+3+\ldots+k}{2}$.

If $S_1^2 + S_2^2 + \ldots + S_{10}^2 = \frac{5}{12}A$. Then A is equal to **[Jan. 12, 2019 (I)]**

(a) 283 (b) 301 (c) 303 (d) 156

113. If the sum of the first 15 terms of the series

$\left(\frac{3}{4}\right)^3 + \left(1\frac{1}{2}\right)^3 + \left(2\frac{1}{4}\right)^3 + 3^3 + \left(3\frac{3}{4}\right)^3 + \ldots$ is equal to 225 k then k is equal to : **[Jan. 12, 2019 (II)]**

(a) 108 (b) 27 (c) 54 (d) 9

114. The sum of the following series **[Jan. 09, 2019 (II)]**

$$1+6+\frac{9(1^2+2^2+3^2)}{7}+\frac{12(1^2+2^2+3^2+4^2)}{9}+\frac{15(1^2+2^2+\ldots+5^2)}{11}+\ldots$$ up to 15 terms, is:

(a) 7520 (b) 7510 (c) 7830 (d) 7820

115. The sum of the first 20 terms of the series

$1+\frac{3}{2}+\frac{7}{4}+\frac{15}{8}+\frac{31}{16}+\ldots$ is? **[Online April 16, 2018]**

(a) $38+\frac{1}{2^{20}}$ (b) $39+\frac{1}{2^{19}}$

(c) $39+\frac{1}{2^{20}}$ (d) $38+\frac{1}{2^{19}}$

116. Let A be the sum of the first 20 terms and B be the sum of the first 40 terms of the series

$1^2 + 2\cdot 2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + \ldots$

If $B - 2A = 100\lambda$, then λ is equal to : **[2018]**

(a) 248 (b) 464 (c) 496 (d) 232

117. Let $a, b, c \in R$. If $f(x) = ax^2 + bx + c$ is such that $a + b + c = 3$ and $f(x + y) = f(x) + f(y) + xy, \forall x, y \in R$, then $\sum_{n=1}^{10} f(n)$ is equal to : **[2017]**

(a) 255 (b) 330 (c) 165 (d) 190

118. Let $S_n = \frac{1}{1^3}+\frac{1+2}{1^3+2^3}+\frac{1+2+3}{1^3+2^3+3^3}+\ldots+\frac{1+2+\ldots+n}{1^3+2^3+\ldots+n^3}$, If $100\,S_n = n$, then n is equal to : **[Online April 9, 2017]**

(a) 199 (b) 99 (c) 200 (d) 19

119. If the sum of the first n terms of the series

$\sqrt{3}+\sqrt{75}+\sqrt{243}+\sqrt{507}+\ldots$ is $435\sqrt{3}$, then n equals : **[Online April 8, 2017]**

(a) 18 (b) 15 (c) 13 (d) 29

120. If the sum of the first ten terms of the series $\left(1\frac{3}{5}\right)^2+\left(2\frac{2}{5}\right)^2+\left(3\frac{1}{5}\right)^2+4^2+\left(4\frac{4}{5}\right)^2+\ldots$, is $\frac{16}{5}m$, then m is equal to : **[2016]**

(a) 100 (b) 99 (c) 102 (d) 101

121. For $x \in R, x \neq -1$, if $(1 + x)^{2016} + x(1 + x)^{2015} + x^2(1 + x)^{2014} + \ldots + x^{2016} = \sum_{i=0}^{2016} a_i x^i$, then a_{17} is equal to : **[Online April 9, 2016]**

(a) $\frac{2017!}{17!2000!}$ (b) $\frac{2016!}{17!\,1999!}$

(c) $\frac{2016!}{16!}$ (d) $\frac{2017!}{2000!}$

122. The sum of first 9 terms of the series. **[2015]**

$\frac{1^3}{1}+\frac{1^3+2^3}{1+3}+\frac{1^3+2^3+3^3}{1+3+5}+\ldots$

(a) 142 (b) 192 (c) 71 (d) 96

123. If $\sum_{n=1}^{5}\frac{1}{n(n+1)(n+2)(n+3)} = \frac{k}{3}$, then k is equal to **[Online April 11, 2015]**

(a) $\frac{1}{6}$ (b) $\frac{17}{105}$ (c) $\frac{55}{336}$ (d) $\frac{19}{112}$

124. The value of $\sum_{r=16}^{30}(r+2)(r-3)$ is equal to : **[Online April 10, 2015]**

(a) 7770 (b) 7785 (c) 7775 (d) 7780

125. If $(10)^9+2(11)^1(10^8)+3(11)^2(10)^7+.....$

$+10(11)^9=k(10)^9$, then k is equal to: **[2014]**

(a) 100 (b) 110 (c) $\frac{121}{10}$ (d) $\frac{441}{100}$

126. The number of terms in an A.P. is even; the sum of the odd terms in it is 24 and that the even terms is 30. If the last term exceeds the first term by $10\frac{1}{2}$, then the number of terms in the A.P. is: **[Online April 19, 2014]**

(a) 4 (b) 8 (c) 12 (d) 16

127. If the sum

$\frac{3}{1^2}+\frac{5}{1^2+2^2}+\frac{7}{1^2+2^2+3^2}+......+$ up to 20 terms is equal to $\frac{k}{21}$, then k is equal to: **[Online April 9, 2014]**

(a) 120 (b) 180 (c) 240 (d) 60

128. The sum of first 20 terms of the sequence 0.7, 0.77, 0.777,....., is **[2013]**

(a) $\frac{7}{81}(179-10^{-20})$ (b) $\frac{7}{9}(99-10^{-20})$

(c) $\frac{7}{81}(179+10^{-20})$ (d) $\frac{7}{9}(99+10^{-20})$

129. The value of $1^2+3^2+5^2+......................+25^2$ is : **[Online April 25, 2013]**

(a) 2925 (b) 1469 (c) 1728 (d) 1456

130. The sum of the series :
$(b)^2+2(d)^2+3(6)^2+...$ upto 10 terms is : **[Online April 23, 2013]**

(a) 11300 (b) 11200 (c) 12100 (d) 12300

131. The sum $\frac{3}{1^2}+\frac{5}{1^2+2^2}+\frac{7}{1^2+2^2+3^2}+....$ upto 11-terms is: **[Online April 22, 2013]**

(a) $\frac{7}{2}$ (b) $\frac{11}{4}$ (c) $\frac{11}{2}$ (d) $\frac{60}{11}$

132. The sum of the series : **[Online April 9, 2013]**

$1+\frac{1}{1+2}+\frac{1}{1+2+3}+.......$ upto 10 terms, is :

(a) $\frac{18}{11}$ (b) $\frac{22}{13}$ (c) $\frac{20}{11}$ (d) $\frac{16}{9}$

133. **Statement-1:** The sum of the series 1 + (1 + 2 + 4) + (4 + 6 + 9) + (9 + 12 + 16) + ... + (361 + 380 + 400) is 8000.

Statement-2: $\sum_{k=1}^{n}\left(k^3-(k-1)^3\right)=n^3$, for any natural number n. **[2012]**

(a) Statement-1 is false, Statement-2 is true.
(b) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for Statement-1.
(c) Statement-1 is true, statement-2 is true; statement-2 is **not** a correct explanation for Statement-1.
(d) Statement-1 is true, statement-2 is false.

134. If the sum of the series $1^2+2.2^2+3^2+2.4^2+5^2+...2.6^2+...$ upto n terms, when n is even, is $\frac{n(n+1)^2}{2}$, then the sum of the series, when n is odd, is **[Online May 26, 2012]**

(a) $n^2(n+1)$ (b) $\frac{n^2(n-1)}{2}$

(c) $\frac{n^2(n+1)}{2}$ (d) $n^2(n-1)$

135. The sum of the series $1+\frac{4}{3}+\frac{10}{9}+\frac{28}{27}+...$ upto n terms is **[Online May 19, 2012]**

(a) $\frac{7}{6}n+\frac{1}{6}-\frac{2}{3.2^{n-1}}$ (b) $\frac{5}{3}n-\frac{7}{6}+\frac{1}{2.3^{n-1}}$

(c) $n+\frac{1}{2}-\frac{1}{2.3^n}$ (d) $n-\frac{1}{3}-\frac{1}{3.2^{n-1}}$

136. The sum of the series

$\frac{1}{1+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+...$

upto 15 terms is **[Online May 12, 2012]**

(a) 1 (b) 2 (c) 3 (d) 4

137. The sum of the series
$1^2+2.2^2+3^2+2.4^2+5^2+2.6^2+....+2(2m)^2$ is **[Online May 7, 2012]**

(a) $m(2m+1)^2$ (b) $m^2(m+2)$
(c) $m^2(2m+1)$ (d) $m(m+2)^2$

138. The sum to infinite term of the series

$1+\frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}+\frac{14}{3^4}+...$ is **[2009]**

(a) 3 (b) 4 (c) 6 (d) 2

139. The sum of series $\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-$ .... upto infinity is **[2007]**

(a) $e^{-\frac{1}{2}}$ (b) $e^{+\frac{1}{2}}$ (c) e^{-2} (d) e^{-1}

140. The sum of the series

$1+\frac{1}{4.2!}+\frac{1}{16.4!}+\frac{1}{64.6!}+$ ....ad inf. is **[2005]**

(a) $\frac{e-1}{\sqrt{e}}$ (b) $\frac{e+1}{\sqrt{e}}$ (c) $\frac{e-1}{2\sqrt{e}}$ (d) $\frac{e+1}{2\sqrt{e}}$

141. The sum of series $\frac{1}{2!}+\frac{1}{4!}+\frac{1}{6!}+$..... is **[2004]**

(a) $\frac{(e^2-2)}{e}$ (b) $\frac{(e-1)^2}{2e}$

(c) $\frac{(e^2-1)}{2e}$ (d) $\frac{(e^2-1)}{2}$

142. The sum of the first n terms of the series

$1^2+2.2^2+3^2+2.4^2+5^2+2.6^2+...$

is $\frac{n(n+1)^2}{2}$ when n is even. When n is odd the sum is **[2004]**

(a) $\left[\frac{n(n+1)}{2}\right]^2$ (b) $\frac{n^2(n+1)}{2}$

(c) $\frac{n(n+1)^2}{4}$ (d) $\frac{3n(n+1)}{2}$

143. If $S_n=\sum_{r=0}^{n}\frac{1}{{}^nC_r}$ and $t_n=\sum_{r=0}^{n}\frac{r}{{}^nC_r}$, then $\frac{t_n}{S_n}$ is equal to **[2004]**

(a) $\frac{2n-1}{2}$ (b) $\frac{1}{2}n-1$ (c) $n-1$ (d) $\frac{1}{2}n$

144. The sum of the series **[2003]**

$\frac{1}{1.2}-\frac{1}{2.3}+\frac{1}{3.4}$........... up to ∞ is equal to

(a) $\log_e\left(\frac{4}{e}\right)$ (b) $2\log_e 2$

(c) $\log_e 2-1$ (d) $\log_e 2$

145. $1^3-2^3+3^3-4^3+...+9^3=$ **[2002]**

(a) 425 (b) –425 (c) 475 (d) –475

146. The value of $2^{1/4}.\,4^{1/8}.\,8^{1/16}\ ...\ \infty$ is **[2002]**

(a) 1 (b) 2 (c) 3/2 (d) 4

Hints & Solutions

1. (c) Let common difference of series

$a_1, a_2, a_3, ..., a_n$ be d.

$\because a_{40} = a_1 + 39d = -159$ ...(i)

and $a_{100} = a_1 + 99d = -399$ ...(ii)

From equations (i) and (ii),

$d = -4$ and $a_1 = -3$

Since, the common difference of $b_1, b_2, ..., b_n$ is 2 more than common difference of $a_1, a_2, ..., a_n$.

$\therefore$ Common difference of $b_1, b_2, b_3, ...$ is (-2).

$\because b_{100} = a_{70}$

$\Rightarrow b_1 + 99(-2) = (-3) + 69(-4)$

$\Rightarrow b_1 = 198 - 279 \Rightarrow b_1 = -81$

2. (a) Given that $3^{2\sin 2\alpha - 1}$, 14, $3^{4 - 2\sin 2\alpha}$ are in A.P.

So, $3^{2\sin 2\alpha - 1} + 3^{4 - 2\sin 2\alpha} = 28$

$$\Rightarrow \frac{3^{2\sin 2\alpha}}{3} + \frac{81}{3^{2\sin 2\alpha}} = 28$$

Let $3^{2\sin 2\alpha} = x$

$$\Rightarrow \frac{x}{3} + \frac{81}{x} = 28$$

$\Rightarrow x^2 - 84x + 243 = 0 \Rightarrow x = 81, x = 3$

When $x = 81 \Rightarrow \sin 2\alpha = 2$ (Not possible)

When $x = 3 \Rightarrow \alpha = \frac{\pi}{12}$

$\therefore a = 3^0 = 1, d = 14 - 1 = 13$

$a_6 = a + 5d = 1 + 65 = 66.$

3. (a) $S = \log_7 x^2 + \log_7 x^3 + \log_7 x^4 + ...20$ terms

$\because S = 460$

$\Rightarrow \log_7 (x^2 \cdot x^3 \cdot x^4 \cdot ......x^{21}) = 460$

$\Rightarrow \log_7 x^{(2+3+4......21)} = 460$

$\Rightarrow (2 + 3 + 4 + ... + 21)\log_7 x = 460$

$$\Rightarrow \frac{20}{2}(2 + 21)\log_7 x = 460$$

$$\Rightarrow \log_7 x = \frac{460}{230} = 2 \Rightarrow x = 7^2 = 49$$

4. (d) Given that $a_1 = 1$ and $a_n = 300$ and $d \in \mathbf{Z}$

$\therefore 300 = 1 + (n-1)d$

$$\Rightarrow d = \frac{299}{(n-1)} = \frac{23 \times 13}{(n-1)},$$

$\because d$ is an integer

$\therefore n - 1 = 13$ or 23

$\Rightarrow n = 14$ or 24 $(\because 15 \le n \le 50)$

$\Rightarrow n = 24$ and $d = 13$

$a_{20} = 1 + 19 \times 13 = 248$

$$s_{20} = \frac{20}{2}(2 + 19 \times 13) = 2490.$$

5. (a) Given $a = 3$ and $S_{25} = S_{40} - S_{25}$

$\Rightarrow 2S_{25} = S_{40}$

$$\Rightarrow 2 \times \frac{25}{2}[6 + 24d] = \frac{40}{2}[6 + 39d]$$

$\Rightarrow 25[6 + 24d] = 20[6 + 39d]$

$\Rightarrow 5(2 + 8d) = 4(2 + 13d)$

$\Rightarrow 10 + 40d = 8 + 52d$

$$\Rightarrow d = \frac{1}{6}$$

6. (b) $S_n = 20 + 19\frac{3}{5} + 19\frac{1}{5} + 18\frac{4}{5} + ...$

$\because S_n = 488$

$$488 = \frac{n}{2}\left[2\left(\frac{100}{5}\right) + (n-1)\left(-\frac{2}{5}\right)\right]$$

$$488 = \frac{n}{2}(101 - n) \Rightarrow n^2 - 101n + 2440 = 0$$

$\Rightarrow n = 61$ or 40

For $n = 40 \Rightarrow T_n > 0$

For $n = 61 \Rightarrow T_n < 0$

$$n^{\text{th}} \text{ term} = T_{61} = \frac{100}{5} + (61 - 1)\left(-\frac{2}{5}\right) = -4$$

7. **(d)** Let common difference be d.

$\because S_{11}=0 \;\therefore \frac{11}{2}\{2a_1+10\cdot d\}=0$

$\Rightarrow a_1+5d=0 \Rightarrow d=-\frac{a_1}{5}$ ...(i)

Now, $S=a_1+a_3+a_5+...+a_{23}$

$=a_1+(a_1+2d)+(a_1+4d)+....+(a_1+22d)$

$=12a_1+2d\frac{11\times 12}{2}$

$=12\left[a_1+11\cdot\left(-\frac{a_1}{5}\right)\right]$ (From (i))

$=12\times\left(-\frac{6}{5}\right)a_1=-\frac{72}{5}a_1$

8. **(14)** First common term of both the series is 23 and common difference is $7\times 4=28$

$\because$ Last term ≤ 407

$\Rightarrow 23+(n-1)\times 28\le 407$

$\Rightarrow (n-1)\times 28\le 384$

$\Rightarrow n\le\frac{384}{28}+1$

$\Rightarrow n\le 14.71$

Hence, $n=14$

9. **(d)** $T_{10}=\frac{1}{20}=a+9d$ ...(i)

$T_{20}=\frac{1}{10}=a+19d$ ...(ii)

Solving equations (i) and (ii), we get

$a=\frac{1}{200}, d=\frac{1}{200}$

$\Rightarrow S_{200}=\frac{200}{2}\left[\frac{2}{200}+\frac{199}{200}\right]=\frac{201}{2}=100\frac{1}{2}$

10. **(b)** If $2^{1-x}+2^{1+x}, f(x), 3^x+3^{-x}$ are in A.P., then

$f(x)=\left(\frac{2^{1+x}+2^{1-x}+3^x+3^{-x}}{2}\right)$

$2f(x)=2\left(2^x+\frac{1}{2^x}\right)+\left(3^x+\frac{1}{3^x}\right)$

Using AM $\ge$ GM

$f(x)\ge 3$

11. **(d)** Let 5 terms of A.P. be

$a-2d, a-d, a, a+d, a+2d$.

Sum $=25 \Rightarrow 5a=25 \Rightarrow a=5$

Product $=2520$

$(5-2d)(5-d)5(5+d)(5+2d)=2520$

$\Rightarrow (25-4d^2)(25-d^2)=504$

$\Rightarrow 625-100d^2-25d^2+4d^4=504$

$\Rightarrow 4d^4-125d^2+625-504=0$

$\Rightarrow 4d^4-125d^2+121=0$

$\Rightarrow 4d^4-121d^2-4d^2+121=0$

$\Rightarrow (d^2-1)(4d^2-121)=0$

$\Rightarrow d=\pm 1, d=\pm\frac{11}{2}$

$d=\pm 1$ and $d=-\frac{11}{2}$, does not give $\frac{-1}{2}$ as a term

$\therefore d=\frac{11}{2}$

$\therefore$ Largest term $=5+2d=5+11=16$

12. **(c)** Given, $S_4=16$ and $S_6=-48$

$\Rightarrow 2(2a+3d)=16\Rightarrow 2a+3d=8$ ...(i)

And $3[2a+5d]=-48\Rightarrow 2a+5d=-16$

$\Rightarrow 2d=-24$ [using equation (i)]

$\Rightarrow d=-12$ and $a=22$

$\therefore S_{10}\frac{10}{2}=(44+9(-12))=-320$

13. **(a)** Let the common difference of the A.P. is 'd'.

Given, $a_1+a_7+a_{16}=40$

$\Rightarrow a_1+a_1+6d+a_1+15d=40$

$\Rightarrow 3a_1+21d=40$

$\Rightarrow a_1+7d=\frac{40}{3}$ ...(i)

Now, sum of first 15 terms of this A.P. is,

$S_{15}=\frac{15}{2}[2a_1+14d]=15(a_1+7d)$

$=15\left(\frac{40}{3}\right)=200$ [Using (i)]

14. **(b)** $a_1+a_4+a_7+....+a_{16}=114$

$\Rightarrow 3(a_1+a_{16})=114$

$\Rightarrow a_1+a_{16}=38$

Now, $a_1+a_6+a_{11}+a_{16}=2(a_1+a_{16})=2\times 38=76$

15. **(d)** $\because S_n=\left(50-\frac{7A}{2}\right)n+n^2\times\frac{A}{2} \Rightarrow a_1=50-3S$

$\therefore d=a_2-a_1=S_{n_2}-S_{n_1}-S_{n_1}\Rightarrow d=\frac{A}{2}\times 2=A$

Now, $a_{50}=a_1+49\times d$

$=(50-3A)+49A=50+46A$

So, $(d, a_{50})=(A, 50+46A)$

16. **(d)** $\because f(x+y) = f(x) . f(y)$

$\Rightarrow$ Let $f(x) = t^x$

$\because f(1) = 2$

$\therefore t' = 2$

$\Rightarrow f(x) = 2^x$

Since, $\sum_{k=1}^{10} f(a+k) = 16(2^{10}-1)$

Then, $\sum_{k=1}^{10} 2^{a+k} = 16(2^{10}-1)$

$\Rightarrow 2^a \sum_{k=1}^{10} 2^k = 16(2^{10}-1)$

$\Rightarrow 2^a \times \frac{((2^{10})-1)\times 2}{(2-1)} = 16\times(2^{10}-1) 2.2^a = 16$

$\Rightarrow a = 3$

17. **(d)** Let three terms of A.P. are $a-d, a, a+d$

Sum of terms is, $a-d+a+a+d = 33 \Rightarrow a = 11$

Product of terms is, $(a-d)\,a\,(a+d) = 11(121-d^2) = 1155$

$\Rightarrow 121 - d^2 = 105 \Rightarrow d = \pm 4$

if $d = 4$

$T_{11} = T_1 + 10d = 7 + 10(4) = 47$

if $d = -4$

$T_{11} = T_1 + 10d = 15 + 10(-4) = -25$

18. **(d)** $\because 91 = 13 \times 7$

Then, the required numbers are either divisible by 7 or 13.

$\therefore$ Sum of such numbers = Sum of no. divisible by 7 + sum of the no. divisible by 13 – Sum of the numbers divisible by 91

$= (105 + 112 + ... + 196) + (104 + 117 + ... + 195) - 182$

$= 2107 + 1196 - 182 = 3121$

19. **(b)** Since ${}^nC_4, {}^nC_5$ and ${}^nC_6$ are in A.P.

$2\,{}^nC_5 = {}^nC_4 + {}^nC_6$

$2 = \frac{{}^nC_4}{{}^nC_5} + \frac{{}^nC_6}{{}^nC_5}$

$2 = \frac{5}{n-4} + \frac{n-5}{5}$

$\Rightarrow 12(n-4) = 30 + n^2 - 9n + 20$

$\Rightarrow n^2 - 21n + 98 = 0$

$(n-7)(n-14) = 0$

$(n-7)(n-14) = 0$

$n = 7, n = 14$

20. **(c)** Let first term and common difference of AP be a and d respectively, then

$t_n = a + (n-1)d$

$\therefore \quad t_{19} = a + 18d = 0$

$\therefore \quad a = -18d \qquad ...(i)$

$\therefore \quad \frac{t_{49}}{t_{29}} = \frac{a+48d}{a+28d}$

$= \frac{-18d+48d}{-18d+28d} = \frac{30d}{10d} = 3$

$t_{49} : t_{29} = 3 : 1$

21. **(d)** Two digit positive numbers which when divided by 7 yield 2 as remainder are 12 terms i.e,16, 23, 30, ..., 93

Two digit positive numbers which when divided by 7 yield 5 as remainder are 13 terms i.e,12, 19, 26, ..., 96

By using AP sum of 16, 23, ..., 93, we get

$S_1 = 16 + 23 + 30 + ... + 93 = 654$

By using AP sum of 12, 19, 26, ..., 96, we get

$S_1 = 12 + 19 + 26 + ... + 96 = 702$

$\therefore$ required Sum $= S_1 + S_2 = 654 + 702 = 1356$

22. **(a)** $S = \sum_{i=1}^{30} a_i = \frac{30}{2}[2a_1 + 29d]$

$T = \sum_{i=1}^{15} a_{(2i-1)} = \frac{15}{2}[2a_1 + 28d]$

Since, $S - 2T = 75$

$\Rightarrow 30\,a_1 + 435d - 30a_1 - 420d = 75$

$\Rightarrow d = 5$

Also, $a_5 = 27 \Rightarrow a_1 + 4d = 27 \Rightarrow a_1 = 7$,

Hence, $a_{10} = a_1 + 9d = 7 + 9 \times 5 = 52$

23. **(c)** $\because \frac{1}{x_1}, \frac{1}{x_2}, \frac{1}{x_3}, ..., \frac{1}{x_n}$ are in A.P

$x_1 = 4$ and $x_{21} = 20$

Let 'd' be the common difference of this A.P

$\therefore$ its 21st term $= \frac{1}{x_{21}} = \frac{1}{x_1} + [(21-1)\times d]$

$\Rightarrow d = \frac{1}{20}\times\left(\frac{1}{20}-\frac{1}{4}\right) \quad \Rightarrow d = -\frac{1}{100}$

Also $\quad x_n > 50$ (given).

$\therefore \frac{1}{x_n} = \frac{1}{x_1} + [(n-1)\times d]$

$\Rightarrow x_n = \frac{x_1}{1+(n-1)\times d\times x_1}$

$\therefore \frac{x_1}{1+(n-1)\times d\times x_1} > 50$

$$\Rightarrow \frac{4}{1+(n-1)\times\left(-\frac{1}{100}\right)\times 4} > 50$$

$$\Rightarrow 1+(n-1)\times\left(-\frac{1}{100}\right)\times 4 < \frac{4}{50}$$

$$\Rightarrow -\frac{1}{100}(n-1) < -\frac{23}{100}$$

$\Rightarrow n-1 > 23 \quad \Rightarrow n > 24$

Therefore, $n = 25$.

$$\Rightarrow \sum_{i=1}^{25}\frac{1}{x_i} = \frac{25}{2}\left[\left(2\times\frac{1}{4}\right)+(25-1)\times\left(-\frac{1}{100}\right)\right] = \frac{13}{4}$$

24. **(a)** Suppose d_1 is the common difference of the A.P. $x_1, x_2, \ldots. x_n$ then

$$\because x_8 - x_3 = 5d_1 = 12 \Rightarrow d_1 = \frac{12}{5} = 2.4$$

$$\Rightarrow x_5 = x_3 + 2d_1 = 8 + 2\times\frac{12}{5} = 12.8$$

Suppose d_2 is the common difference of the A.P $\frac{1}{h_1}, \frac{1}{h_2}, \ldots.. \frac{1}{h_n}$ then

$$5d_2 = \frac{1}{20} - \frac{1}{8} = \frac{-3}{40} \Rightarrow d_2 = \frac{-3}{200}$$

$$\because \frac{1}{h_{10}} = \frac{1}{h_7} + 3d_2 = \frac{1}{200} \Rightarrow h_{10} = 200$$

$$\Rightarrow x_5 . h_{10} = 12.8\times 200 = 2560$$

25. **(b)** $\because \sum_{k=0}^{12} a_{4k+1} = 416 \Rightarrow \frac{13}{2}[2a_1 + 48d] = 416$

$\Rightarrow a_1 + 24d = 32$ ...(i)

Now, $a_9 + a_{43} = 66 \Rightarrow 2a_1 + 50d = 66$ ...(ii)

From eq. (i) & (ii) we get; $d = 1$ and $a_1 = 8$

Also, $\sum_{r=1}^{17} a_r^2 = \sum_{r=1}^{17}[8+(r-1)1]^2 = 140\text{ m}$

$$\Rightarrow \sum_{r=1}^{17}(r+7)^2 = 140\text{ m}$$

$$\Rightarrow \sum_{r=1}^{17}(r^2+14r+49) = 140\text{ m}$$

$$\Rightarrow \left(\frac{17\times18\times35}{6}\right)+14\left(\frac{17\times18}{2}\right)+(49\times17) = 140$$

$\Rightarrow m = 34$

26. **(c)** We have

$9(25a^2+b^2)+25(c^2-3ac) = 15b(3a+c)$

$\Rightarrow 225a^2+9b^2+25c^2-75ac = 45ab+15bc$

$\Rightarrow (15a)^2+(3b)^2+(5c)^2-75ac-45ab-15bc = 0$

$$\frac{1}{2}[(15a-3b)^2+(3b-5c)^2+(5c-15a)^2] = 0$$

It is possible when $15a - 3b = 0$, $3b - 5c = 0$ and $5c - 15a = 0$

$\Rightarrow 15a = 3b \Rightarrow b = 5a$

$$\Rightarrow b = \frac{5c}{3}, a = \frac{c}{3}$$

$$\Rightarrow a+b = \frac{c}{3}+\frac{5c}{3} = \frac{6c}{3}$$

$\Rightarrow a+b = 2c$

$\Rightarrow$ b, c, a are in A.P.

27. **(a)** **By Arithmetic Mean**:

$a + c = 2b$

Consider $a = b = c = 2$

$\Rightarrow abc = 8$

$\Rightarrow a + b = 2b$

$\therefore$ minimum possible value of $b = 2$

28. **(a)** $a_3 + a_7 + a_{11} + a_{15} = 72$

$(a_3 + a_{15}) + (a_7 + a_{11}) = 72$

$a_3 + a_{15} + a_7 + a_{11} = 2(a_1 + a_{17})$

$a_1 + a_{17} = 36$

$$S_{17} = \frac{17}{2}[a_1 + a_{17}] = 17\times18 = 306$$

29. **(b)** Let p, q, r are in AP

$\Rightarrow 2q = p + r$ ...(i)

Given $\frac{1}{\alpha}+\frac{1}{\beta} = 4$

$$\Rightarrow \frac{\alpha+\beta}{\alpha\beta} = 4$$

We have $\alpha+\beta = -q/p$ and $\alpha\beta = \frac{r}{p}$

$$\Rightarrow \frac{-\frac{q}{p}}{\frac{r}{p}} = 4 \Rightarrow q = -4r \qquad \text{....(ii)}$$

From (i), we have

$2(-4r) = p + r$

$p = -9r$ ....(iii)

Now, $|\alpha-\beta| = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$

$$= \sqrt{\left(\frac{-q}{p}\right)^2 - \frac{4r}{p}} = \frac{\sqrt{q^2-4pr}}{|p|}$$

From (ii) and (iii)

$$= \frac{\sqrt{16r^2+36r^2}}{|-9r|} = \frac{2\sqrt{13}}{9}$$

30. **(b)** Given n = 20; S_{20} = ?
Series (1) → 3, 7, <u>11</u>, 15, 19, 23, 27, <u>31</u>, 35, 39, 43, 47, <u>51</u>, 55, 59...
Series (2) → 1, 6, <u>11</u>, 16, 21, 26, <u>31</u>, 36, 41, 46, <u>51</u>, 56, 61, 66, 71.
The common terms between both the series are 11, 31, 51, 71...
Above series forms an Arithmetic progression (A.P).
Therefore, first term (a) = 11 and
common difference (d) = 20

Now, $S_n = \frac{n}{2}[2a+(n-1)d]$

$S_{20} = \frac{20}{2}$ [2 × 11 + (20 – 1) 20]
S_{20} = 10 [22 + 19 × 20]
S_{20} = 10 × 402 = 4020
∴ S_{20} = 4020

31. **(c)** Let a be the first term and d be the common difference of given A.P.
Second term, $a + d = 12$ ...(i)
Sum of first nine terms,

$$S_9 = \frac{9}{2}(2a+8d) = 9(a+4d)$$

Given that S_9 is more than 200 and less than 220
$\Rightarrow 200 < S_9 < 220$
$\Rightarrow 200 < 9(a+4d) < 220$
$\Rightarrow 200 < 9(a+d+3d) < 220$
Putting value of $(a+d)$ from equation (i)
$200 < 9(12+3d) < 220$
$\Rightarrow 200 < 108+27d < 220$
$\Rightarrow 200-108 < 108+27d-108 < 220-108$
$\Rightarrow 92 < 27d < 112$
Possible value of d is 4
27 × 4 = 108
Thus, 92 < 108 < 112
Putting value of d in equation (i)
$a + d = 12$
$a = 12 - 4 = 8$
4^{th} term $= a + 3d = 8 + 3 \times 4 = 20$

32. **(c)** If d be the common difference, then
$m = a_4 - a_7 + a_{10} = a_4 - a_7 + a_7 + 3d = a_7$

$$S_{13} = \frac{13}{2}[a_1 + a_{13}] = \frac{13}{2}[a_1 + a_7 + 6d]$$

$$= \frac{13}{2}[2a_7] = 13a_7 = 13\text{ m}$$

33. **(b)** Given $S_n = 2n + 3n^2$
Now, first term $= 2 + 3 = 5$
second term $= 2(2) + 3(4) = 16$
third term $= 2(3) + 3(9) = 33$
Now, sum given in option (b) only has the same first term and difference between 2nd and 1st term is double also.

34. **(b)** $\frac{a_1+a_2+a_3+......+a_p}{a_1+a_2+a_3+......+a_q} = \frac{p^3}{q^3}$

$\Rightarrow \frac{a_1+a_2}{a_1} = \frac{8}{1} \Rightarrow a_1 + (a_1+d) = 8a_1$

$\Rightarrow d = 6a_1$

Now $\frac{a_6}{a_{21}} = \frac{a_1+5d}{a_1+20d}$

$$= \frac{a_1+5\times 6a_1}{a_1+20\times 6a_1} = \frac{1+30}{1+120} = \frac{31}{121}$$

35. **(d)** Let 'a' is the first term and 'd' is the common difference of an $A.P.$
Now, According to the question
$100a_{100} = 50a_{50}$
$100(a+99d) = 50(a+49d)$
$\Rightarrow 2a + 198d = a + 49d \Rightarrow a + 149d = 0$
Hence, $T_{150} = a + 149d = 0$

36. **(d)** Given : $\frac{a_p+a_q}{2} = \frac{a_r+a_s}{2}$
$\Rightarrow a + (p-1)d + a + (q-1)d$
$= a + (r-1)d + a + (s-1)d$
$\Rightarrow 2a + (p+q)d - 2d = 2a + (r+s)d - 2d$
$\Rightarrow (p+q)d = (r+s)d \Rightarrow p+q = r+s.$

37. **(a)** Since, $\sec(\theta-\phi)$, $\sec\theta$ and $\sec(\theta+\phi)$ are in A.P.,

$\therefore 2\sec\theta = \sec(\theta-\phi) + \sec(\theta+\phi)$

$$\Rightarrow \frac{2}{\cos\theta} = \frac{\cos(\theta+\phi)+\cos(\theta-\phi)}{\cos(\theta-\phi)\cos(\theta+\phi)}$$

$$\Rightarrow 2\left(\cos^2\theta - \sin^2\phi\right) = \cos\theta\left[2\cos\theta\cos\phi\right]$$

$$\Rightarrow \cos^2\theta(1-\cos\phi) = \sin^2\phi = 1-\cos^2\phi$$

$$\Rightarrow \cos^2\theta = 1+\cos\phi = 2\cos^2\frac{\phi}{2}$$

$$\therefore \cos\theta = \sqrt{2}\cos\frac{\phi}{2}$$

But given $\cos\theta = k\cos\frac{\phi}{2}$

$\therefore k = \sqrt{2}$

38. **(b)** Let A.P. be $a, a+d, a+2d, ...$

$a_2 + a_4 + ... + a_{200} = \alpha$

$\Rightarrow \frac{100}{2}\left[2(a+d)+(100-1)2d\right]=\alpha$ ...(i)

and $a_1 + a_3 + a_5 + ... + a_{199} = \beta$

$\Rightarrow \frac{100}{2}[2a+(100-1)2d]=\beta$ ...(ii)

Subtracting (ii) from (i), we get

$d = \frac{\alpha-\beta}{100}$

39. **(c)** Let number of months $= n$

$\therefore 200 \times 3 + (240 + 280 + 320 + ... + (n-3)^{th} \text{ term}) = 11040$

$\Rightarrow \frac{n-3}{2}[2\times 240+(n-4)\times 40] = 11040-600$

$\Rightarrow (n-3)[240+20n-80] = 10440$

$\Rightarrow (n-3)(20n+160) = 10440$

$\Rightarrow (n-3)(n+8) = 522$

$\Rightarrow n^2 + 5n - 546 = 0$

$\Rightarrow (n+26)(n-21) = 0$

$\therefore n = 21$

40. **(a)** Till 10^{th} minute number of counted notes $= 1500$

Remaining notes $= 4500 - 1500 = 3000$

$3000 = \frac{n}{2}[2\times 148+(n-1)(-2)] = n[148-n+1]$

$n^2 - 149n + 3000 = 0$

$\Rightarrow n = 125, 24$

But $n = 125$ is not possible

$\therefore$ Total time $= 24 + 10 = 34$ minutes.

41. **(d)** Given that

$$\frac{S_p}{S_q} = \frac{\frac{p}{2}[2a_1+(p-1)d]}{\frac{q}{2}[2a_1+(q-1)d]} = \frac{p^2}{q^2}$$

$$\Rightarrow \frac{2a_1+(p-1)d}{2a_1+(q-1)d} = \frac{p}{q}$$

Put $p = 11$ and $q = 41$

$$\frac{a_1+5d}{a_1+20d} = \frac{11}{41} \Rightarrow \frac{a_6}{a_{21}} = \frac{11}{41}$$

42. **(c)** Coeficient of r^{th}, $(r+1)^{th}$ and $(r+2)^{th}$ terms is ${}^mC_{r-1}$, ${}^mC_r$ and ${}^mC_{r+1}$ resp.

Given that ${}^mC_{r-1}$, ${}^mC_r$, ${}^mC_{r+1}$ are in A.P.

$2\,{}^mC_r = {}^mC_{r-1} + {}^mC_{r+1}$

$$\Rightarrow 2 = \frac{{}^mC_{r-1}}{{}^mC_r} + \frac{{}^mC_{r+1}}{{}^mC_r} = \frac{r}{m-r+1} + \frac{m-r}{r+1}$$

$\Rightarrow m^2 - m(4r+1) + 4r^2 - 2 = 0$.

43. **(d)** $T_m = a + (m-1)d = \frac{1}{n}$ .....(i)

$T_n = a + (n-1)d = \frac{1}{m}$ .....(ii)

Subtracting (ii) from (i), we get

$(m-n)d = \frac{1}{n} - \frac{1}{m} \Rightarrow d = \frac{1}{mn}$

From (i) $a = \frac{1}{mn} \Rightarrow a - d = 0$

44. **(b)** $1, \log_9(3^{1-x}+2), \log_3(4.3^x - 1)$ are in A.P.

$\because$ a, b, c are in A.P then $b = a + c$

$\Rightarrow 2\log_9(3^{1-x}+2) = 1 + \log_3(4.3^x-1)$

$\because$ $\log_{b^q} a^p = \frac{p}{q}\log_b a$

$\Rightarrow \log_3(3^{1-x}+2) = \log_3 3 + \log_3(4.3^x - 1)$

$\Rightarrow \log_3(3^{1-x}+2) = \log_3[3(4.3^x-1)]$

$\Rightarrow 3^{1-x} + 2 = 3(4.3^x - 1)$

$\Rightarrow 3.3^{-x} + 2 = 12.3^x - 3.$

Put $3^x = t$

$\Rightarrow \frac{3}{t} + 2 = 12t - 3 \Rightarrow 12t^2 - 5t - 3 = 0;$

Hence $t = -\frac{1}{3}, \frac{3}{4}$

$\Rightarrow 3^x = \frac{3}{4}$ (as $3^x \neq -ve$)

$\Rightarrow x = \log_3\left(\frac{3}{4}\right)$ or $x = \log_3 3 - \log_3 4$

$\Rightarrow x = 1 - \log_3 4$

45. **(d)** Let $f(1) = k$, then $f(2) = f(1+1) = k^2$

$f(3) = f(2+1) = k^3$

$$\sum_{x=1}^{\infty} f(x) = 2 \Rightarrow k + k^2 + k^3 + ...\infty = 2$$

$\Rightarrow \frac{k}{1-k} = 2 \Rightarrow k = \frac{2}{3}$

Now, $\frac{f(4)}{f(2)} = \frac{k^4}{k^2} = k^2 = \frac{4}{9}.$

46. **(c)** Rearrange given equation, we get

$(a^2p^2 - 2abp + b^2) + (b^2p^2 - 2bcp + c^2)$
$+ (c^2p^2 - 2cdp + d^2) = 0$

$\Rightarrow (ap-b)^2 + (bp-c)^2 + (cp-d)^2 = 0$

$\therefore ap - b = bp - c = cp - d = 0$

$\Rightarrow \frac{b}{a} = \frac{c}{b} = \frac{d}{c}$ $\therefore$ a, b, c, d are in G.P.

47. **(5.00)**

$\because f(x+y)=f(x)\cdot f(y) \quad \forall x\in R$ and $f(1)=3$

$\Rightarrow f(x)=3^x \Rightarrow f(i)=3^i$

$\Rightarrow \sum_{i=1}^{n} f(i)=363$

$\Rightarrow 3+3^2+3^3+....+3^n=363$

$\Rightarrow \frac{3(3^n-1)}{3-1}=363 \quad \left[\because S_n=\frac{a(r^n-1)}{(r-1)}\right]$

$\Rightarrow 3^n-1=\frac{363\times 2}{3}=242$

$\Rightarrow 3^n=243=3^5 \Rightarrow n=5$

48. **(b)** Given sequence are in G.P. and common ratio $\frac{3}{2}$

$\therefore \frac{2^{10}\left(\left(\frac{3}{2}\right)^{11}-1\right)}{\left(\frac{3}{2}-1\right)}=S-2^{11}$

$\Rightarrow 2^{10}\frac{\left(\frac{3^{11}-2^{11}}{2^{11}}\right)}{\frac{1}{2}}=S-2^{11}$

$\Rightarrow 3^{11}-2^{11}=S-2^{11} \Rightarrow S=3^{11}$

49. **(b)** Let the first term be 'a' and common ratio be 'r'.

$\because ar(1+r+r^2)=3$ ...(i)

and $ar^5(1+r+r^2)=243$ ...(ii)

From (i) and (ii),

$r^4=81 \Rightarrow r=3$ and $a=\frac{1}{13}$

$\therefore S_{50}=\frac{a(r^{50}-1)}{r-1}=\frac{3^{50}-1}{26} \quad \left[\because S_n=\frac{a(r^n-1)}{(r-1)}\right]$

50. **(b)** Let $\alpha, \beta, \gamma, \delta$ be in G.P., then $\alpha\delta=\beta\gamma$

$\Rightarrow \frac{\alpha}{\beta}=\frac{\gamma}{\delta} \Rightarrow \left|\frac{\alpha-\beta}{\alpha+\beta}\right|=\left|\frac{\gamma-\delta}{\gamma+\delta}\right|$

$\Rightarrow \frac{\sqrt{9-4p}}{3}=\frac{\sqrt{36-4q}}{6}$

$\Rightarrow 36-16p=36-4q \Rightarrow q=4p$

$\therefore \frac{2q+p}{2q-p}=\frac{8p+p}{8p-p}=\frac{9p}{7p}=\frac{9}{7}$

51. **(4)**

$(0.16)^{\log_{2.5}\left(\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+.....\infty\right)}$

$=0.16^{\log_{2.5}\left(\frac{\frac{1}{3}}{1-\frac{1}{3}}\right)} \quad \left[\because S_\infty=\frac{a}{1-r}\right]$

$=0.16^{\log_{2.5}\left(\frac{1}{2}\right)}$

$=(2.5)^{-2\log_{2.5}\left(\frac{1}{2}\right)}=\left(\frac{1}{2}\right)^{-2}=4.$

52. **(c)** Let terms of G.P. be $\frac{a}{r}, a, ar$

$\therefore a\left(\frac{1}{r}+1+r\right)=S$ ...(i)

and $a^3=27$

$\Rightarrow a=3$ ...(ii)

Put $a=3$ in eqn. (1), we get

$S=3+3\left(r+\frac{1}{r}\right)$

If $f(x)=x+\frac{1}{x}$, then $f(x)\in(-\infty,-2]\cup[2,\infty)$

$\Rightarrow 3f(x)\in(-\infty,-6]\cup[6,\infty)$

$\Rightarrow 3+3f(x)\in(-\infty,-3]\cup[9,\infty)$

Then, it concludes that

$S\in(-\infty,-3]\cup[9,\infty)$

53. **(c)** $S=(x+y)+(x^2+y^2+xy)+(x^3+x^2y+xy^2+y^3)+....\infty$

$=\frac{1}{x-y}\left[(x^2-y^2)+(x^3-y^3)+(x^4-y^4+....\infty\right]$

$=\frac{1}{x-y}\left[\frac{x^2}{1-x}-\frac{y^2}{1-y}\right]=\frac{(x-y)(x+y-xy)}{(x-y)(1-x)(1-y)}$

$\left[\because S_\infty=\frac{a}{1-r}\right]$

$=\frac{x+y-xy}{(1-x)(1-y)}$

54. **(c)** $S=(x+x^2+x^3+....9 \text{ terms})$

$+a[k+(k+2)++(k+4)+...9 \text{ terms}]$

$\Rightarrow S=\frac{x(x^9-1)}{x-1}+\frac{9}{2}[2ak+8\times(2a)]$

$\Rightarrow S = \frac{x^{10}-x}{x-1} + \frac{9a(k+8)}{1} = \frac{x^{10}-x+45a(x-1)}{x-1}$ (Given)

$\Rightarrow \frac{x^{10}-x+9a(k+8)(x-1)}{x-1} = \frac{x^{10}-x+45a(x-1)}{x-1}$

$\Rightarrow 9a(k+8) = 45a \Rightarrow k+8 = 5 \Rightarrow k = -3.$

55. **(a)** $2^{\frac{1}{4}+\frac{2}{16}+\frac{3}{48}+.....\infty}$

$= 2^{\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+.....\infty} = \sqrt{2}$

56. **(d)** Let G.P. be a, ar, ar^2 .....

$\sum_{n=1}^{100} a_{2n+1} = a_3 + a_5 + ... + a_{201} = 200$

$\Rightarrow \frac{ar^2(r^{200}-1)}{r^2-1} = 200$ ...(i)

$\sum_{n=1}^{100} a_{2n} = a_2 + a_4 + ... + a_{200} = 100$

$\Rightarrow \frac{ar(r^{200}-1)}{r^2-1} = 100$ ...(ii)

From equations (i) and (ii), $r = 2$ and
$a_2 + a_3 + ... + a_{200} + a_{201} = 300$
$\Rightarrow r(a_1 + ... + a_{200}) = 300$

$\Rightarrow \sum_{n=1}^{200} a_n = \frac{300}{r} = 150$

57. **(b)** $y = 1 + \cos^2\theta + \cos^4\theta + ...$

$\Rightarrow y = \frac{1}{1-\cos^2\theta} \Rightarrow \frac{1}{y} = \sin^2\theta$

$x = 1 - \tan^2\theta + \tan^4\theta + ...$

$x = \frac{1}{1-(-\tan^2\theta)} = \frac{1}{\sec^2\theta} \Rightarrow x = \cos^2\theta$

$y = \frac{1}{\sin^2\theta} \Rightarrow y = \frac{1}{1-x}$

$\therefore y(1-x) = 1$

58. **(b)** $\frac{(49)^{126}-1}{48} = \frac{((49)^{63}+1)(49^{63}-1)}{48} \left[\because S_n = \frac{a(r^n-1)}{r-1}\right]$

$\therefore$ K = 63

59. **(b)** Since, $a_1 + a_2 = 4 \Rightarrow a_1 + a_1 r = 4$ ...(i)

$a_3 + a_4 = 16 \qquad \Rightarrow a_1 r^2 + a_1 r^3 = 16$ ...(ii)

From eqn. (i), $a_1 = \frac{4}{1+r}$ and substituting the value of a_1, in eqn (ii),

$\left(\frac{4}{1+r}\right)^{r^2} + \left(\frac{4}{1+r}\right)^{r^3} = 16$

$\Rightarrow 4r^2(1+r) = 16(1+r)$

$\Rightarrow r^2 = 4 \qquad \therefore r = \pm 2$

$r = 2, a_1(1+2) = 4 \Rightarrow a_1 = \frac{4}{3}$

$r = -2, a_1(1-2) = 4 \Rightarrow a_1 = -4$

$\sum_{i=1}^{a} a_i = \frac{a_1(r^q-1)}{r-1} = \frac{(-4)((-2)^9-1)}{-2-1}$

$= \frac{4}{3}(-513) = 4\lambda \Rightarrow \lambda = -171$

60. **(b)** The given series is in G.P. then

$S_n = \frac{a(1-r^n)}{1-r}$

$\frac{(1+x)^{10}\left[1-\left(\frac{x}{1+x}\right)^{11}\right]}{\left(1-\frac{x}{1+x}\right)}$

$\Rightarrow \frac{(1+x)^{10}\left[(1+x)^{11}-x^{11}\right]}{(1+x)^{11} \times \frac{1}{(1+x)}} = (1+x)^{11} - x^{11}$

$\therefore$ Coefficient of x^7 is ${}^{11}C_7 = {}^{11}C_{11-7} = {}^{11}C_4 = 330$

61. **(d)** $\because$ α, β, γ are three consecutive terms of a non-constant G.P.

$\therefore \beta^2 = \alpha\gamma$

So roots of the equation $\alpha x^2 + 2\beta x + \gamma = 0$ are

$\frac{-2\beta \pm 2\sqrt{\beta^2 - \alpha\gamma}}{2\alpha} = \frac{\beta}{\alpha}$

$\because \alpha x^2 + 2\beta x + \gamma = 0$ and $x^2 + x - 1 = 0$ have a common root.

$\therefore$ this root satisfy the equation $x^2 + x - 1 = 0$

$\beta^2 - \alpha\beta - \alpha^2 = 0$

$\Rightarrow \alpha\gamma - \alpha\beta - \alpha^2 = 0 \Rightarrow \alpha + \beta = \gamma$

Now, $\alpha(\beta + \gamma) = \alpha\beta + \alpha\gamma$

$= \alpha\beta + \beta^2 = (\alpha + \beta)\beta = \beta\gamma$

62. **(d)** $\because a, b, c$ are in G.P. $\Rightarrow b = ar, c = ar^2$

$\because 3a, 7b, 15c$ are in A.P. $\Rightarrow 3a, 7ar, 15ar^2$ are in A.P.

$\therefore 14ar = 3a + 15ar^2$

$\Rightarrow 15r^2 - 14r + 3 = 0 \Rightarrow r = \frac{1}{3}$ or $\frac{3}{5}$

$\because r < \frac{1}{2} \quad \therefore r = \frac{3}{5}$ rejected

Fourth term $= 15ar^2 + 7ar - 3a$

$= a(15r^2 + 7r - 3) = a\left(\frac{15}{9} + \frac{7}{3} - 3\right) = a$

63. **(a)** Since a, b, c are in G.P.

$\therefore\ b^2 = ac$

Given equation is, $ax^2 + 2bx + c = 0$

$\Rightarrow ax^2 + 2\sqrt{ac}x + c = 0 \Rightarrow (\sqrt{a}x + \sqrt{c})^2 = 0$

$\Rightarrow x = -\sqrt{\frac{c}{a}}$

Also, given that $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ have a common root.

$\Rightarrow x = -\sqrt{\frac{c}{a}}$ must satisfy $dx^2 + 2ex + f = 0$

$\Rightarrow d.\frac{c}{a} + 2e\left(-\sqrt{\frac{c}{a}}\right) + f = 0$

$\frac{d}{a} - \frac{2e}{\sqrt{ac}} + \frac{f}{c} = 0 \Rightarrow \frac{d}{a} - \frac{2e}{b} + \frac{f}{c} = 0$

$\Rightarrow \frac{2e}{b} = \frac{d}{a} + \frac{f}{c} \Rightarrow \frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P.

64. **(d)** Let three terms of a G.P. be $\frac{a}{r}, a, ar$

$\frac{a}{r} \cdot a \cdot ar = 512$

$a^3 = 512 \Rightarrow a = 8$

4 is added to each of the first and the second of three terms then three terms are, $\frac{8}{r} + 4, 8 + 4, 8r$.

$\because \frac{8}{r} + 4, 12, 8r$ form an A.P.

$\therefore\ 2 \times 12 = \frac{8}{r} + 8r + 4$

$\Rightarrow\ 2r^2 - 5r + 2 = 0$

$\Rightarrow\ (2r - 1)(r - 2) = 0$

$\Rightarrow\ r = \frac{1}{2}$ or 2

Therefore, sum of three terms $= \frac{8}{2} + 8 + 16 = 28$

65. **(c)** $x^2 \sin\theta - x(\sin\theta \,.\, \cos\theta + 1) + \cos\theta = 0.$

$x^2 \sin\theta - x\sin\theta \,.\, \cos\theta - x + \cos\theta = 0.$

$x\sin\theta\,(x - \cos\theta) - 1\,(x - \cos\theta) = 0.$

$(x - \cos\theta)(x\sin\theta - 1) = 0.$

$\therefore\ x = \cos\theta, \text{cosec}\,\theta, \theta \in (0, 45°)$

$\therefore\ \alpha = \cos\theta, \beta = \text{cosec}\,\theta$

$\sum_{n=0}^{\infty} \alpha^n = 1 + \cos\theta + \cos^2\theta + \ldots \infty = \frac{1}{1 - \cos\theta}$

$\sum_{n=0}^{\infty} \frac{(-1)^n}{\beta^n} = 1 - \frac{1}{\text{cosec}\,\theta} + \frac{1}{\text{cosec}^2\theta} - \frac{1}{\text{cosec}^3\theta} + \ldots \infty$

$= 1 - \sin\theta + \sin^2\theta - \sin^3\theta + \ldots \infty.$

$= \frac{1}{1 + \sin\theta}$

$\therefore\ \sum_{n=0}^{\infty}\left(\alpha^n + \frac{(-1)^n}{\beta^n}\right) = \sum_{n=0}^{\infty} \alpha^n + \sum_{n=0}^{\infty} \frac{(-1)^n}{\beta^n}$

$= \frac{1}{1 - \cos\theta} + \frac{1}{1 + \sin\theta}.$

66. **(a)** Let $a_1 = a, a_2 = ar, a_3 = ar^2 \ldots a_{10} = ar^9$

where r = common ratio of given G.P.

Given, $\frac{a_3}{a_1} = 25$

$\Rightarrow\ \frac{ar^2}{a} = 25$

$\Rightarrow\ r = \pm 5$

Now, $\frac{a_9}{a_5} = \frac{ar^8}{ar^4} = r^4 = (\pm 5)^4 = 5^4$

67. **(b)** Let the terms of infinite series are $a, ar, ar^2, ar^3, \ldots$

So, $\frac{a}{1 - r} = 3$

Since, sum of cubes of its terms is $\frac{27}{19}$ that is sum of a^3, $a^3r^3, \ldots \infty$ is $\frac{27}{19}$

So, $\frac{a^3}{1 - r^3} = \frac{27}{19}$

$\Rightarrow\ \frac{a}{1 - r} \times \frac{a^2}{(1 + r^2 + r)} = \frac{27}{19}$

$\Rightarrow\ \frac{9(1 + r^2 - 2r) \times 3}{1 + r^2 + r} = \frac{27}{19}$

$\Rightarrow\ 6r^2 - 13r + 6 = 0$

$\Rightarrow\ (3r - 2)(2r - 3) = 0$

$\Rightarrow\ r = \frac{2}{3}, \text{ or } \frac{3}{2}$

As $|r| < 1$

So, $r = \frac{2}{3}$

68. **(d)** $S_n=\left(\frac{1-q^{n+1}}{1-q}\right), T_n=\frac{1-\left(\frac{q+1}{2}\right)^{n+1}}{1-\left(\frac{q+1}{2}\right)}$

$$\Rightarrow T_{100}=\frac{1-\left(\frac{q+1}{2}\right)^{101}}{1-\left(\frac{q+1}{2}\right)}$$

$$Sn=\frac{1}{1-q}-\frac{q^{n+1}}{1-q}, T_{100}=\frac{2^{101}-(q+1)^{101}}{2^{100}(1-q)}$$

Now, ${}^{101}C_1+{}^{101}C_2 S_1+{}^{101}C_3 S_2+\ldots+{}^{101}C_{101} S_{100}$

$$=\left(\frac{1}{1-q}\right)({}^{101}C_2+\ldots+{}^{101}C_{101})$$

$$-\frac{1}{1-q}({}^{101}C_2 q^2+{}^{101}C_3 q^3+\ldots+{}^{101}C_{101} q^{101})+101$$

$$=\frac{1}{1-q}(2^{101}-1-101)-\left(\frac{1}{1-q}\right)((1+q)^{101}-1-{}^{101}C_1 q)+101$$

$$=\frac{1}{1-q}[2^{101}-102-(1+q)^{101}+1+101q]+101$$

$$=\frac{1}{1-q}[2^{101}-101+101q-(1+q)^{101}]+101$$

$$=\left(\frac{1}{1-q}\right)[2^{101}-(1+q)^{101}]=2^{100}T_{100}$$

Hence, by comparison $\alpha=2^{100}$

69. **(d)** Let first term and common difference be A and D respectively.

$\therefore a=A+6D, b=A+10D$

and $c=A+12D$

Since, a, b, c are in G.P.

Hence, relation between a, b and c is,

$\therefore b^2=a.c.$

$\therefore (A+10D)^2=(A+6D)(A+12D)$

$\therefore 14D+A=0$

$\therefore A=-14D$

$\therefore a=-8D, b=-4D$ and $c=-2D$

$$\therefore \frac{a}{c}=\frac{-8D}{-2D}=4$$

70. **(d)** $\because a, b, c,$ are in G.P.

$\Rightarrow b^2=ac$

Since, $a+b+c=xb$

$\Rightarrow a+c=(x-1)b$

Take square on both sides, we get

$a^2+c^2+2ac=(x-1)^2 b^2$

$\Rightarrow a^2+c^2=(x-1)^2 ac-2ac\ [\because b^2=ac]$

$\Rightarrow a^2+c^2=ac[(x-1)^2-2]$

$\Rightarrow a^2+c^2=ac[x^2-2x-1]$

$\because a^2+c^2$ are positive and $b^2=ac$ which is also positive.

Then, x^2-2x-1 would be positive but for $x=2$, x^2-2x-1 is negative.

Hence, x cannot be taken as 2.

71. **(c)** First term $=b$ and common ratio $=$ r

For infinite series, Sum $=\frac{b}{1-r}=5$

$\Rightarrow b=5(1-r)$

So, interval of $b=(0, 10)$ as, $-1<r<1$ for infinite G.P.

72. **(b)** $A_n=\left(\frac{3}{4}\right)-\left(\frac{3}{4}\right)^2+\left(\frac{3}{4}\right)^3-\ldots+(-1)^{n-1}\left(\frac{3}{4}\right)^n$

Which is a G.P. with $a=\frac{3}{4}, r=\frac{-3}{4}$ and number of terms $=n$

$$\therefore A_n=\frac{\frac{3}{4}\times\left(1-\left(\frac{-3}{4}\right)^n\right)}{1-\left(\frac{-3}{4}\right)}=\frac{\frac{3}{4}\times\left(1-\left(\frac{-3}{4}\right)^n\right)}{\frac{7}{4}}$$

$$\Rightarrow A_n=\frac{3}{7}\left[1-\left(\frac{-3}{4}\right)^n\right] \quad \ldots(i)$$

As, $B_n=1-A_n$

For least odd natural number p, such that $B_n>A_n$

$\Rightarrow 1-A_n>A_n \quad \Rightarrow 1>2\times A_n \ \Rightarrow A_n<\frac{1}{2}$

From eqn. (i), we get

$$\frac{3}{7}\times\left[1-\left(\frac{-3}{4}\right)^n\right]<\frac{1}{2}\Rightarrow 1-\left(\frac{-3}{4}\right)^n<\frac{7}{6}$$

$$\Rightarrow 1-\frac{7}{6}<\left(\frac{-3}{4}\right)^n \Rightarrow \frac{-1}{6}<\left(\frac{-3}{4}\right)^n$$

As n is odd, then $\left(\frac{-3}{4}\right)^n = -\frac{3^n}{4}$

So $\frac{-1}{6} < -\left(\frac{3}{4}\right)^n \Rightarrow \frac{1}{6} > \left(\frac{3}{4}\right)^n$

$\log\left(\frac{1}{6}\right) = n\log\left(\frac{3}{4}\right) \Rightarrow 6.228 < n$

Hence, n should be 7.

73. **(d)** $\because a, b, c$ are in A.P. then

$a + c = 2b$

also it is given that,

$a + b + c = \frac{3}{4}$ ...(i)

$\Rightarrow 2b + b = \frac{3}{4} \Rightarrow b = \frac{1}{4}$ ...(ii)

Again it is given that, a^2, b^2, c^2 are in G.P. then

$(b^2)^2 = a^2c^2 \Rightarrow ac = \pm\frac{1}{16}$ ...(iii)

From (i), (ii) and (iii), we get;

$a \pm \frac{1}{16a} = \frac{1}{2} \Rightarrow 16a^2 - 8a \pm 1 = 0$

Case I: $16a^2 - 8a + 1 = 0$

$\Rightarrow a = \frac{1}{4}$ (not possible as $a < b$)

Case II: $16a^2 - 8a - 1 = 0 \Rightarrow a = \frac{8 \pm \sqrt{128}}{32}$

$\Rightarrow a = \frac{1}{4} \pm \frac{1}{2\sqrt{2}}$

$\therefore a = \frac{1}{4} - \frac{1}{2\sqrt{2}}$ $(\because a < b)$

74. **(d)** Let the GP be a, ar and ar^2 then $a = A + d$; $ar = A + 4d$; $ar^2 = A + 8d$

$\Rightarrow \frac{ar^2 - ar}{ar - a} = \frac{(A+8d)-(A+4d)}{(A+4d)-(A+d)}$

$r = \frac{4}{3}$

75. **(b)** $z = 1 + ai$

$z^2 = 1 - a^2 + 2ai$

$z^2.z = \{(1-a^2) + 2ai\}$ $\{1 + ai\}$

$= (1-a^2) + 2ai + (1-a^2)$ $ai - 2a^2$

$\because$ z^3 is real $\Rightarrow 2a + (1-a^2)\,a = 0$

$a(3-a^2) = 0 \Rightarrow a = \sqrt{3}$ $(a > 0)$

$1 + z + z^2 .... z^{11} = \frac{z^{12}-1}{z-1} = \frac{(1+\sqrt{3}i)^{12} - 1}{1+\sqrt{3}i - 1}$

$= \frac{(1+\sqrt{3}i)^{12} - 1}{\sqrt{3}i}$

$(1+\sqrt{3}i)^{12} = 2^{12}\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)^{12}$

$= 2^{12}\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)^{12} = 2^{12}(\cos 4\pi + i\sin 4\pi) = 2^{12}$

$\Rightarrow \frac{2^{12}-1}{\sqrt{3}i} = \frac{4095}{\sqrt{3}i} = -\frac{4095}{3}\sqrt{3}i = -1365\sqrt{3}i$

76. **(d)** $m = \frac{l+n}{2}$ and common ratio of G.P. $= r = \left(\frac{n}{l}\right)^{\frac{1}{4}}$

$\therefore$ $G_1 = l^{3/4}n^{1/4}, G_2 = l^{1/2}n^{1/2}, G_3 = l^{1/4}n^{3/4}$

$G_1^4 + 2G_2^4 + G_3^4 = l^3n + 2l^2n^2 + ln^3$

$= ln(l+n)^2 = ln \times (2m)^2 = 4lm^2n$

77. **(d)** Let a, ar and ar^2 be the first three terms of G.P

According to the question

$a(ar)(ar^2) = 1000 \Rightarrow (ar)^3 = 1000 \Rightarrow ar = 10$

and $ar^2 + ar^3 = 60 \Rightarrow ar(r + r^2) = 60$

$\Rightarrow r^2 + r - 6 = 0$

$\Rightarrow r = 2, -3$

$a = 5, a = -\frac{10}{3}$ (reject)

Hence, $T_7 = ar^6 = 5(2)^6 = 5 \times 64 = 320.$

78. **(b)** Let a, ar, ar^2 are in G.P.

According to the question

$a, 2ar, ar^2$ are in A.P.

$\Rightarrow$ $2 \times 2ar = a + ar^2$

$\Rightarrow$ $4r = 1 + r^2 \Rightarrow r^2 - 4r + 1 = 0$

$r = \frac{4 \pm \sqrt{16-4}}{2} = 2 \pm \sqrt{3}$

Since $r > 1$

$\therefore$ $r = 2 - \sqrt{3}$ is rejected

Hence, $r = 2 + \sqrt{3}$

79. **(b)** $1 - \frac{2}{3} - \frac{2}{3^2} .... \frac{2}{3^{n-1}} < \frac{1}{100}$

$\Rightarrow 1 - \frac{2}{3}\left[\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + ... \frac{1}{3^{n-1}}\right] < \frac{1}{100}$

$\Rightarrow \frac{1 - 2\left[\frac{1}{3}\left(\frac{1}{3^n} - 1\right)\right]}{\frac{1}{3} - 1} < \frac{1}{100}$

$\Rightarrow 1-2\left[\frac{3^n-1}{2.3^n}\right]<\frac{1}{100}$

$\Rightarrow 1-\left[\frac{3^n-1}{3^n}\right]<\frac{1}{100}$

$\Rightarrow 1-1+\frac{1}{3^n}<\frac{1}{100} \Rightarrow 100 < 3^n$

Thus, least value of n is 5

80. **(c)** According to Question

$\Rightarrow \frac{S_5}{S_5'} = 49$ {here, S_5 = Sum of first 5 terms and S_5 = Sum of their reciprocals)

$\Rightarrow \frac{\frac{a(r^5-1)}{(r-1)}}{\frac{a^{-1}(r^{-5}-1)}{(r^{-1}-1)}} = 49$

$\Rightarrow \frac{a(r^5-1)\times(r^{-1}-1)}{a^{-1}(r^{-5}-1)\times(r-1)} = 49$

or $\frac{a^2(1-r^5)\times(1-r)\times r^5}{(1-r^5)\times(1-r)\times r} = 49$

$\Rightarrow a^2r^4 = 49 \Rightarrow a^2r^4 = 7^2$

$\Rightarrow \boxed{ar^2 = 7}$ ...(i)

Also, given, $S_1 + S_3 = 35$

$a + ar^2 = 35$ ...(ii)

Now substituting the value of eq. (i) in eq. (ii)

$a + 7 = 35$

$\boxed{a = 28}$

81. **(d)** Let given expansion be

$S = (1+x)^{1000} + x(1+x)^{999} + x^2(1+x)^{998} + \ldots + \ldots + x^{1000}$

Put $1 + x = t$

$S = t^{1000} + xt^{999} + x^2(t)^{998} + \ldots + x^{1000}$

This is a G.P with common ratio $\frac{x}{t}$

$S = \frac{t^{1000}\left[1-\left(\frac{x}{t}\right)^{1001}\right]}{1-\frac{x}{t}}$

$= \frac{(1+x)^{1000}\left[1-\left(\frac{x}{1+x}\right)^{1001}\right]}{1-\frac{x}{1+x}}$

$= \frac{(1+x)^{1001}\left[(1+x)^{1001}-x^{1001}\right]}{(1+x)^{1001}}$

$= [(1+x)^{1001} - x^{1001}]$

Now coeff of x^{50} in above expansion is equal to coeff of x^{50} in $(1+x)^{1001}$ which is ${}^{1001}C_{50}$

$= \frac{(1001)!}{50!(951)!}$

82. **(b)** Let a, b, c, d be four numbers of the sequence.

Now, according to the question $b^2 = ac$ and $c - b = 6$ and $a - c = 6$

Also, given $\boxed{a = d}$

$\therefore\ b^2 = ac \Rightarrow b^2 = a\left[\frac{a+b}{2}\right]$ $(\because\ 2c = a + b)$

$\Rightarrow a^2 - 2b^2 + ab = 0$

Now, $c - b = 6$ and $a - c = 6$,

gives $a - b = 12$

$\Rightarrow b = a - 12$

$\therefore a^2 - 2b^2 + ab = 0$

$\Rightarrow a^2 - 2(a-12)^2 + a(a-12) = 0$

$\Rightarrow a^2 - 2a^2 - 288 + 48a + a^2 - 12a = 0$

$\Rightarrow 36a = 288 \Rightarrow a = 8$

Hence, last term is $d = a = 8$.

83. **(d)** The given relation can be written as

$(a^2p^2 - 2abp + b^2) + (b^2p^2 + c^2 - 2bpc) + (c^2p^2 + d^2 - 2pcd) \le 0$

or $(ap-b)^2 + (bp-c)^2 + (cp-d)^2 \le 0$ ...(i)

Since a, b, c, d and p are all real, the inequality (i) is possible only when each of factor is zero.

i.e., $ap - b = 0$, $bp - c = 0$ and $cp - d = 0$

or $p = \frac{b}{a} = \frac{c}{b} = \frac{d}{c}$

or a, b, c, d are in G.P.

84. **(c)** Let $a, ar, ar^2, ar^3, ar^4, ar^5$ be six terms of a G.P. where 'a' is first term and r is common ratio.

According to given conditions, we have

$ar^3 - a = 5 \Rightarrow a(r^3-1) = 52$ ...(i)

and $a + ar + ar^2 = 26$

$\Rightarrow a(1 + r + r^2) = 26$ ...(ii)

To find: $a(1 + r + r^2 + r^3 + r^4 + r^5)$

Consider

$a[1 + r + r^2 + r^3 + r^4 + r^5]$

$= a[1 + r + r^2 + r^3(1 + r + r^2)]$

$= a[1+r+r^2][1+r^3]$ ...(iii)

Divide (i) by (ii), we get

$\frac{r^3-1}{1+r+r^2} = 2$,

we know $r^3 - 1 = (r-1)(1+r+r^2)$

$\therefore\ r-1 = 2 \Rightarrow r = 3$ and $a = 2$

$\therefore\ a(1+r+r^2+r^3+r^4+r^5) = a(1+r+r^2)(1+r^3)$

$= 2(1+3+9)(1+27) = 26 \times 28 = 728$

85. **(b)** ATQ,

$a + ar = 12$ ...(i)

$ar^2 + ar^3 = 48$ ...(ii)

$\Rightarrow \frac{ar^2(1+r)}{a(1+r)} = \frac{48}{12} \Rightarrow r^2 = 4, \Rightarrow r = -2$

($\because$ terms are alternately + ve and –ve)

$\Rightarrow a = -12$

86. **(b)** Let the series $a, ar, ar^2, ...$ are in geometric progression.

Given that, $a = ar + ar^2$

$\Rightarrow 1 = r + r^2 \qquad \Rightarrow r^2 + r - 1 = 0$

$\Rightarrow r = \frac{-1 \pm \sqrt{1-4\times -1}}{2} = \frac{-1\pm\sqrt{5}}{2}$

$\Rightarrow r = \frac{\sqrt{5}-1}{2}$ [$\because$ terms of G.P. are positive

$\therefore$ r should be positive]

87. **(d)** $\sum_{k=1}^{10}\left(\sin\frac{2k\pi}{11} + i\cos\frac{2k\pi}{11}\right)$

$= i\sum_{k=1}^{10}\left(\cos\frac{2k\pi}{11} - i\sin\frac{2k\pi}{11}\right)$ [$\because e^{i\theta} = \cos\theta + i\sin\theta$]

$= i\sum_{k=1}^{10} e^{-\frac{2k\pi}{11}i} = i\left\{\sum_{k=0}^{10} e^{-\frac{2k\pi}{11}i} - 1\right\}$

$= i\left[1 + e^{-\frac{2\pi}{11}i} + e^{-\frac{4\pi}{11}i} + ...11 \text{ terms}\right] - i$

$= i\left[\frac{1-\left(e^{-\frac{2\pi}{11}}\right)^{11}}{1-e^{-\frac{2\pi}{11}i}}\right] - i = i\left[\frac{1-e^{-2\pi i}}{1-e^{-\frac{2\pi}{11}i}}\right] - i$

$= i \times 0 - i$ [$\because e^{-2\pi i} = 1$]

$= -i$

88. **(d)** $(1-ax)^{-1}(1-bx)^{-1}$

$= (1 + ax + a^2x^2 + ...)(1 + bx + b^2x^2 + ...)$

$\therefore$ Coefficient of x^n

$x^n = b^n + ab^{n-1} + a^2b^{n-2} + ... + a^{n-1}b + a^n$

{which is a G.P. with $r = \frac{a}{b}$

$\therefore$ Its sum is $= \frac{b^n\left[1-\left(\frac{a}{b}\right)^{n+1}\right]}{1-\frac{a}{b}}$}

$= \frac{b^{n+1}-a^{n+1}}{b-a} \qquad \therefore\ a_n = \frac{b^{n+1}-a^{n+1}}{b-a}$

89. **(b)** Let two numbers be a and b then $\frac{a+b}{2} = 9$

$\Rightarrow a + b = 18$ and $\sqrt{ab} = 4 \Rightarrow ab = 16$

$\therefore$ Equation with roots a and b is

$x^2 - (a+b)x + ab = 0 \Rightarrow x^2 - 18x + 16 = 0$

90. **(b)** Let a = first term of G.P. and r = common ratio of G.P.;

Then G.P. is a, ar, ar^2

Given $S_\infty = 20 \Rightarrow \frac{a}{1-r} = 20$

$\Rightarrow a = 20(1-r)$ ... (i)

Also $a^2 + a^2r^2 + a^2r^4 + ...$ to $\infty = 100$

$\Rightarrow \frac{a^2}{1-r^2} = 100 \Rightarrow \frac{[20(1-r)]^2}{1-r^2} = 100$ [from (i)]

$\Rightarrow \frac{400(1-r)^2}{(1-r)(1+r)} = 100 \Rightarrow 4(1-r) = 1 + r$

$\Rightarrow 1 + r = 4 - 4r \Rightarrow 5r = 3 \Rightarrow r = 3/5$.

91. **(b)** $\because a_4 = 2 \Rightarrow ar^4 = 2$

Now, $a \times ar \times ar^2 \times ar^3 \times ar^4 \times ar^5 \times ar^6 \times ar^7 \times ar^8$

$= a^9 r^{36} = (ar^4)^9 = 2^9 = 512$

92. **(c)** $\sin^2 2\theta + \cos^4 2\theta = \frac{3}{4}$

$\Rightarrow 1 - \cos^2 2\theta + \cos^4 2\theta = \frac{3}{4}$

$\Rightarrow \cos^2 2\theta(1 - \cos^2 2\theta) = \frac{1}{4}$ ...(i)

$\because$ G.M. $\le$ A.M.

$\therefore (\cos^2 2\theta)(1-\cos^2 2\theta) \le \left(\frac{\cos^2 2\theta + (1-\cos^2 2\theta)}{2}\right)^2$

$= \frac{1}{4}$ ...(ii)

So, from equation (i) and (ii), we get.

G.M. = A.M.
It is possible only if,
$\cos^2 2\theta = 1 - \cos^2 2\theta$

$\Rightarrow \cos^2 2\theta = \frac{1}{2} \Rightarrow \cos 2\theta = \pm\frac{1}{\sqrt{2}}$

$\Rightarrow \theta = \frac{\pi}{8}, \frac{3\pi}{8} \quad \therefore \text{ Sum} = \frac{\pi}{8} + \frac{3\pi}{8} = \frac{\pi}{2}$

93. (39)

Let m arithmetic mean be $A_1, A_2 \ldots A_m$ and G_1, G_2, G_3 be geometric mean.
The A.P. formed by arithmetic mean is,
$3, A_1, A_2, A_3, \ldots\ldots A_m, 243$

$\therefore d = \frac{243-3}{m+1} = \frac{240}{m+1}$

The G.P. formed by geometric mean
$3, G_1, G_2, G_3, 243$

$r = \left(\frac{243}{3}\right)^{\frac{1}{3+1}} = (81)^{1/4} = 3$

$\because A_4 = G_2$

$\Rightarrow 3 + 4\left(\frac{240}{m+1}\right) = 3(3)^2$

$\Rightarrow 3 + \frac{960}{m+1} = 27 \Rightarrow m+1 = 40 \Rightarrow m = 39.$

94. (d) A.T.Q.,
A.M. = 5 G.M.

$\frac{a+b}{2} = 5\sqrt{ab}$

$\frac{a+b}{\sqrt{ab}} = 10$

$\therefore \frac{a}{b} = \frac{10+\sqrt{96}}{10-\sqrt{96}} = \frac{10+4\sqrt{6}}{10-4\sqrt{6}}$

Use Componendo and Dividendo

$\frac{a+b}{a-b} = \frac{20}{8\sqrt{6}} = \frac{5}{2\sqrt{6}} = \frac{5\sqrt{6}}{12}$

95. (b) $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{y}{1 - \tan A \tan B}$ where $y = \tan A + \tan B$

$\Rightarrow \tan A \tan B = 1 - \sqrt{3}y$

Also AM $\geq$ GM

$\Rightarrow \frac{\tan A + \tan B}{2} \geq \sqrt{\tan A \tan B}$

$\Rightarrow y \geq 2\sqrt{1-\sqrt{3}y}$

$\Rightarrow y^2 \geq 4 - 4\sqrt{3}y$

$\Rightarrow y^2 + 4\sqrt{3}y - 4 \geq 0$

$\Rightarrow y \leq -2\sqrt{3} - 4$ or $y \geq -2\sqrt{3} + 4$

($y \leq -2\sqrt{3} - 4$ is not possible as $\tan A \tan B > 0$)

96. (b) $x + y + z = 12$
AM $\geq$ GM

$\frac{3\left(\frac{x}{3}\right) + 4\left(\frac{y}{4}\right) + 5\left(\frac{z}{5}\right)}{12} \geq \sqrt[12]{\left(\frac{x}{3}\right)^3\left(\frac{y}{4}\right)^4\left(\frac{z}{5}\right)^5}$

$\frac{x^3y^4z^5}{3^3 4^4 5^5} \leq 1$

$x^3y^4z^5 \leq 3^3.4^4.5^5$
$x^3y^4z^5 \leq (0.1)(600)^3$
But, given $x^3y^4z^5 = (0.1)(600)^3$
$\therefore$ all the number are equal

$\therefore \frac{x}{3} = \frac{y}{4} = \frac{z}{5} (= k)$

$x = 3k; y = 4k; z = 5k$
$x + y + z = 12$
$3k + 4k + 5k = 12$
$k = 1$
$\therefore x = 3; y = 4; z = 5$
$\therefore x^3 + y^3 + z^3 = 216$

97. (a) $G = \sqrt{ab}$

$M = \frac{\frac{1}{a} + \frac{1}{b}}{2}$

$M = \frac{a+b}{2ab}$

Given that $\frac{1}{M} : G = 4 : 5$

$\frac{2ab}{(a+b)\sqrt{ab}} = \frac{4}{5}$

$\Rightarrow \frac{a+b}{2\sqrt{ab}} = \frac{5}{4}$

$\Rightarrow \frac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}} = \frac{5+4}{5-4}$

{Using Componendo & Dividendo}

$$\Rightarrow \frac{(\sqrt{a})^2+(\sqrt{b})^2+2\sqrt{ab}}{(\sqrt{a})^2+(\sqrt{b})^2-2\sqrt{ab}} = \frac{9}{1}$$

$$\Rightarrow \left(\frac{\sqrt{b}+\sqrt{a}}{\sqrt{b}-\sqrt{a}}\right)^2 = \frac{9}{1} \Rightarrow \frac{\sqrt{b}+\sqrt{a}}{\sqrt{b}-\sqrt{a}} = \frac{3}{1}$$

$$\Rightarrow \frac{\sqrt{b}+\sqrt{a}+\sqrt{b}-\sqrt{a}}{\sqrt{b}+\sqrt{a}-\sqrt{b}+\sqrt{a}} = \frac{3+1}{3-1}$$ {Using Componendo & Dividendo}

$$\sqrt{\frac{b}{a}} = \frac{4}{2} = 2$$

$$\frac{b}{a} = \frac{4}{1}$$

$$\frac{a}{b} = \frac{1}{4} \Rightarrow a : b = 1 : 4$$

98. **(d)** $\because a_1, a_2, a_3.....a_n$ are in H.P.

$\therefore \frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}....\frac{1}{a_n}$ are in A.P.

$$\therefore \frac{1}{a_2}-\frac{1}{a_1} = \frac{1}{a_3}-\frac{1}{a_2} = ...= \frac{1}{a_n}-\frac{1}{a_{n-1}} = d \text{ (say)}$$

Then $a_1a_2 = \frac{a_1-a_2}{d}$, $a_2a_3 = \frac{a_2-a_3}{d}$,

$..........., a_{n-1}a_n = \frac{a_{n-1}-a_n}{d}$

Adding all equations, we get

$$\therefore a_1a_2+a_2a_3+.........+a_{n-1}a_n$$

$$= \frac{a_1-a_2}{d}+\frac{a_2-a_3}{d}+....+\frac{a_{n-1}-a_n}{d}$$

$$= \frac{1}{d}[a_1-a_2+a_2-a_3+....+a_{n-1}-a_n] = \frac{a_1-a_n}{d}$$

Also, $\frac{1}{a_n} = \frac{1}{a_1}+(n-1)d$

$$\Rightarrow \frac{a_1-a_n}{a_1a_n} = (n-1)d$$

$$\Rightarrow \frac{a_1-a_n}{d} = (n-1)a_1a_n$$

Which is the required result.

99. **(d)** $x = \sum_{n=0}^{\infty} a^n = \frac{1}{1-a} \Rightarrow a = 1-\frac{1}{x}$

$$y = \sum_{n=0}^{\infty} b^n = \frac{1}{1-b} \Rightarrow b = 1-\frac{1}{y}$$

$$z = \sum_{n=0}^{\infty} c^n = \frac{1}{1-c} \Rightarrow c = 1-\frac{1}{z}$$

a, b, c are in A.P. $\Rightarrow 2b = a+c$

$$2\left(1-\frac{1}{y}\right) = 1-\frac{1}{x}+1-\frac{1}{z}$$

$\frac{2}{y} = \frac{1}{x}+\frac{1}{z} \Rightarrow x, y, z$ are in H.P.

100. **(d)** $ax^2+bx+c=0,\ \alpha+\beta = \frac{-b}{a},\ \alpha\beta = \frac{c}{a}$

ATQ, $\alpha+\beta = \frac{1}{\alpha^2}+\frac{1}{\beta^2}$

$$\alpha+\beta = \frac{\alpha^2+\beta^2}{\alpha^2\beta^2} \Rightarrow -\frac{b}{a} = \frac{\frac{b^2}{a^2}-\frac{2c}{a}}{\frac{c^2}{a^2}}$$

On simplification $2a^2c = ab^2+bc^2$

$\Rightarrow \frac{2a}{b} = \frac{c}{a}+\frac{b}{c}$ [Divide both side by abc]

$\Rightarrow \frac{c}{a}, \frac{a}{b}, \frac{b}{c}$ are in A.P.

$\therefore \frac{a}{c}, \frac{b}{a}, \& \frac{c}{b}$ are in H.P.

101. **(b)** The given series is

$$1+(1-2^2\cdot 1)+(1-4^2\cdot 3)+(1-6^2\cdot 5)+...(1-20^2\cdot 19)$$

$$S = 1+\sum_{r=1}^{10}[1-(2r)^2(2r-1)]$$

$$= 1+\sum_{r=1}^{10}(1-8r^3+4r^2) = 1+10-\sum_{r=1}^{10}(8r^3-4r^2)$$

$$= 11-8\left(\frac{10\times 11}{2}\right)^2+4\times\left(\frac{10\times 11\times 21}{6}\right)$$

$$= 11-2\times(110)^2+4\times 55\times 7$$

$$= 11-220(110-7)$$

$$= 11-220\times 103 = \alpha-220\beta$$

$$\Rightarrow \alpha = 11,\ \beta = 103$$

$$\therefore (\alpha, \beta) = (11, 103)$$

102. **(a)** Given : $f(x+y)=f(x)+f(y), \forall x, y \in R, f(1)=2$

$\Rightarrow f(2)=f(1)+f(1)=2+2=4$

$f(3)=f(1)+f(2)=2+4=6$

$f(n-1)=2(n-1)$

Now, $g(n)=\sum_{k=1}^{n-1} f(k)$

$=f(1)+f(2)+f(3)+....f(n-1)$

$=2+4+6+.....+2(n-1)$

$=2[1+2+3+.....+(n-1)]$

$=2\times\frac{(n-1)(n)}{2}=n^2-n$

$\because g(n)=20$ (given)

So, $n^2-n=20$

$\Rightarrow n^2-n-20=0$

$\Rightarrow (n-5)(n+4)=0$

$\Rightarrow n=5$ or $n=-4$ (not possible)

103. **(504)** $\left[\sum_{n=1}^{7}\frac{n(n+1)(2n+1)}{4}\right]\frac{1}{4}\left[\sum_{n=1}^{7}(2n^3+3n^2+n)\right]$

$=\frac{1}{4}\left[2\left(\frac{7.8}{2}\right)^2+3\left(\frac{7.8.15}{6}\right)+\frac{7.8}{2}\right]$

$\Rightarrow \frac{1}{4}[2\times49\times16+28\times15+28]$

$=\frac{1}{4}[1568+420+28]=504$

104. **(1540)** Given series can be written as

$\sum_{k=1}^{20}\frac{k(k+1)}{2}=\frac{1}{2}\sum_{k=1}^{20}(k^2+k)$

$=\frac{1}{2}\left[\frac{20(21)(41)}{6}+\frac{20(21)}{2}\right]$

$=\frac{1}{2}\left[\frac{420\times41}{6}+\frac{20\times21}{2}\right]=\frac{1}{2}[2870+210]=1540$

105. **(a)** $S=\underbrace{3+4}+\underbrace{8+9}+\underbrace{13+14}+\underbrace{18+19}.....40$ terms

$S=7+17+27+37+47+.....20$ terms

$S_{40}=\frac{20}{2}[2\times7+(19)10]=10[14+190]$

$=10[2040]=(102)(20)$

$\Rightarrow m=20$

106. **(b)** $\because [x]+\left[x+\frac{1}{n}\right]+\left[x+\frac{2}{n}\right]....\left[x+\frac{n-1}{n}\right]=[nx]$

and $[x]+[-x]=-1\ (x\notin z)$

$\therefore \left[-\frac{1}{3}\right]+\left[-\frac{1}{3}-100\right]+...+\left[-\frac{1}{3}-\frac{99}{100}\right]$

$=-100-\left\{\left[\frac{1}{3}\right]+\left[\frac{1}{3}+\frac{1}{100}\right]+...\left[\frac{1}{3}+\frac{99}{100}\right]\right\}$

$=-100-\left[\frac{100}{3}\right]=-133$

107. **(c)** r^{th} term of the series,

$T_r=\frac{(2r+1)(1^3+2^3+3^3+...+r^3)}{1^2+2^2+3^2+...+r^2}$

$T_r=(2r+1)\left(\frac{r(r+1)}{2}\right)^2\times\frac{6}{r(r+1)(2r+1)}=\frac{3r(r+1)}{2}$

$\therefore$ sum of 10 terms is $=S=\sum_{r=1}^{10}T_r=\frac{3}{2}\sum_{r=1}^{10}(r^2+r)$

$=\frac{3}{2}\left\{\frac{10\times(10+1)(2\times10+1)}{6}+\frac{10\times11}{2}\right\}$

$=\frac{3}{2}\times5\times11\times8=660$

108. **(a)** Let, $S=1+\frac{1^3+2^3}{1+2}+\frac{1^3+2^3+3^3}{1+2+3}+...15$ terms

$T_n=\frac{1^3+2^3+...n^3}{1+2+...n}=\frac{\left(\frac{n(n+1)}{2}\right)^2}{\frac{n(n+1)}{2}}=\frac{n(n+1)}{2}$

Now, $S=\frac{1}{2}\left(\sum_{n=1}^{15}n^2+\sum_{n=1}^{15}n\right)=\frac{1}{2}\left(\frac{15(16)(31)}{6}+\frac{15(16)}{2}\right)$

$=680$

$\therefore$ required sum is, $680-\frac{1}{2}\frac{15(16)}{2}=680-60=620$

109. **(b)** $1+2.3+3.5+4.7+........$

Let, $S=(2.3+3.5+4.7+.......)$

Now, $S_{10}=\sum_{n=1}^{10}(n+1)(2n+1)=\sum_{n=1}^{10}\left(2n^2+3n+1\right)$

$$=\frac{2n(n+1)(2n+1)}{6}+\frac{3n(n+1)}{2}+n$$

Put $n=10$

$$=\frac{2.10.11.21}{6}+\frac{3.10.11}{2}+10=945$$

Hence required sum of the series $=1+945=946$

110. (d) Number of balls used in equilateral triangle

$$=\frac{n(n+1)}{2}$$

$\because$ side of equilateral triangle has n-balls

$\therefore$ no. of balls in each side of square is $=(n-2)$

According to the question,

$$\frac{n(n+1)}{2}+99=(n-2)^2$$

$\Rightarrow n^2+n+198=2n^2-8n+8$

$\Rightarrow n^2-9n-190=0 \Rightarrow (n-19)(n+10)=0$

$\Rightarrow n=19$

Number of balls used to form triangle

$$=\frac{n(n+1)}{2}=\frac{19\times 20}{2}=190$$

111. (c) Let, $S=\sum_{k=1}^{20} k.\frac{1}{2^k}$

$$S=\frac{1}{2}+2.\frac{1}{2^2}+3.\frac{1}{2^3}+....+20.\frac{1}{2^{20}} \quad ...(i)$$

$$\frac{1}{2}S=\frac{1}{2^2}+2.\frac{1}{2^3}+....+19\frac{1}{2^{20}}+20\frac{1}{2^{21}} \quad ...(ii)$$

On subtracting equations (ii) by (i),

$$\frac{S}{2}=\left(\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+....+\frac{1}{2^{20}}\right)-20\frac{1}{2^{21}}$$

$$=\frac{\frac{1}{2}\left(1-\frac{1}{2^{20}}\right)}{1-\frac{1}{2}}-20.\frac{1}{2^{21}}=1-\frac{1}{2^{20}}-10.\frac{1}{2^{20}}$$

$$\frac{S}{2}=1-11.\frac{1}{2^{20}} \Rightarrow S=2-11.\frac{1}{2^{19}}=2-\frac{11}{2^{19}}$$

112. (c) $\because 1+2+3+...+k=\frac{k(k+1)}{2}$

$$\therefore S_k=\frac{k(k+1)}{2k}=\frac{k+1}{2}$$

$$\Rightarrow \frac{5}{12}A=\frac{1}{4}[2^2+3^3+...+11^2]$$

$$=\frac{1}{4}[1^2+2^2+...+11^2-1]$$

$$=\frac{1}{4}\left[\frac{11(11+1)(2\times 11+1)}{6}-1\right]$$

$$\frac{1}{4}\left[\frac{11\times 12\times 23}{6}-1\right]$$

$$=\frac{1}{4}[505]$$

$$A=\frac{505}{4}\times\frac{12}{5}=303$$

113. (b) $S=\left(\frac{3}{4}\right)^3+\left(\frac{3}{2}\right)^3+\left(\frac{9}{4}\right)^3+(3)^3+...$

$$S=\left(\frac{3}{4}\right)^3+\left(\frac{6}{4}\right)^3+\left(\frac{9}{4}\right)^3+\left(\frac{12}{4}\right)^3+...$$

Let the general term of S be

$T_r=\left(\frac{3r}{4}\right)^3$, then

$$255K=\sum_{r=1}^{15}T_r=\left(\frac{3}{4}\right)^3\sum_{r=1}^{15}r^3$$

$$255K=\frac{27}{64}\times\left(\frac{15\times 16}{2}\right)^2$$

$\Rightarrow K=27$

114. (d) $S=1+6+\frac{9(1^2+2^2+3^2)}{7}+\frac{12(1^2+2^2+3^2+4^2)}{9}+\frac{15(1^2+2^2+3^2+4^2+5^2)}{11}+...$

$$S=\frac{3\cdot(1)^2}{3}+\frac{6\cdot(1^2+2^2)}{5}+\frac{9\cdot(1^2+2^2+3^2)}{7}+\frac{12\cdot(1^2+2^2+3^2+4^2)}{9}+...$$

Now, n^{th} term of the series,

$$t_n=\frac{3n\cdot(1^2+2^2+...+n^2)}{(2n+1)}$$

$$\Rightarrow t_n=\frac{3n\cdot n(n+1)(2n+1)}{6(2n+1)}=\frac{n^3+n^2}{2}$$

$$\therefore S_n=\Sigma t_n=\frac{1}{2}\left\{\left(\frac{n(n+1)}{2}\right)^2+\frac{n(n+1)(2n+1)}{6}\right\}$$

$$=\frac{n(n+1)}{4}\left(\frac{n(n+1)}{2}+\frac{2n+1}{3}\right)$$

Hence, sum of the series upto 15 terms is,

$$S_{15}=\frac{15\times16}{4}\left\{\frac{15\cdot16}{2}+\frac{31}{3}\right\}$$

$$=60\times120+60\times\frac{31}{3}$$

$$=7200+620=7820$$

115. (d) The general term of the given series $=\frac{2\times2^r-1}{2^r}$, where $r\geq0$

$\therefore$ req. sum $=1+\sum_{r=1}^{19}\frac{2\times2^r-1}{2^r}$

Now, $\sum_{r=1}^{19}\left(\frac{2\times2^r-1}{2^r}\right)=\sum_{r=1}^{19}\left(2-\frac{1}{2^r}\right)$

$$=2(19)-\frac{\frac{1}{2}\left(1-\left(\frac{1}{2}\right)^{19}\right)}{1-\frac{1}{2}}=38+\frac{\left(\frac{1}{2}\right)^{19}-1}{1}$$

$$=38+\left(\frac{1}{2}\right)^{19}-1=37+\left(\frac{1}{2}\right)^{19}$$

$\therefore$ req. sum $=1+37+\left(\frac{1}{2}\right)^{19}=38+\left(\frac{1}{2}\right)^{19}$

116. (a) Here, $B-2A$

$$=\sum_{n=1}^{40}a_n-2\sum_{n=1}^{20}a_n=\sum_{n=21}^{40}a_n-2\sum_{n=1}^{20}a_n$$

$B-2A=(21^2+2.22^2+23^2+2.24^2+.....+40^2)$
$\qquad-(1^2+2.2^2+3^2+2.4^2.....+20^2)$

$=20\,[22+2.24+26+2.28+.....+60]$

$$=20\Big[\underbrace{(22+24+26....60)}_{20\text{ terms}}+\underbrace{(24+28+....+60)}_{10\text{ terms}}\Big]$$

$$20\left[\frac{20}{2}(22+60)+\frac{10}{2}(24+60)\right]$$

$=10[20.82+10.84]$

$=100[164+84]=100.248$

117. (b) $f(x)=ax^2+bx+c$

$f(1)=a+b+c=3\Rightarrow f(1)=3$

Now $f(x+y)=f(x)+f(y)+xy$ ...(i)

Put $x=y=1$ in eqn (i)

$f(2)=f(1)+f(1)+1=2f(1)+1$

$f(2)=7$

$\Rightarrow f(3)=12$

$S_n=3+7+12+.........+t_n$

$S_n=\quad 3+7+12+.........+t_{n-1}+t_n$

$\overline{0=3+4+5\text{ ....to n term}-t_n}$

$t_n=3+4+5+\text{.... upto n terms}$

$$t_n=\frac{(n^2+5n)}{2}$$

$$S_n=\sum t_n=\sum\frac{(n^2+5n)}{2}$$

$$S_n=\frac{1}{2}\left[\frac{n(n+1)(2n+1)}{6}+\frac{5n(n+1)}{2}\right]$$

$$=\frac{n(n+1)(n+8)}{6}$$

$$S_{10}=\frac{10\times11\times18}{6}=330$$

118. (a) $T_n=\dfrac{\frac{n(n+1)}{2}}{\left(\frac{n(n+1)}{2}\right)^2}$

$$\Rightarrow T_n=\frac{2}{n(n+1)}$$

$$\Rightarrow S_n=\sum T_n=2\sum_{n=1}^{n}\left(\frac{1}{n}-\frac{1}{n+1}\right)=2\left\{1-\frac{1}{n+1}\right\}$$

$$\Rightarrow \boxed{S_n=\frac{2n}{n+1}}$$

$\because\ 100\,S_n=n$

$\Rightarrow 100\times\frac{2n}{n+1}=n$

$\Rightarrow n+1=200$

$\Rightarrow n=199$

119. (b) $\because$

$$\sqrt{3}\left[1+\sqrt{25}+\sqrt{81}+\sqrt{69}+......\right]=435\sqrt{3}$$

$$\Rightarrow \sqrt{3}\,[1+5+9+13+.....+T_n]=435\sqrt{3}$$

$$\Rightarrow \sqrt{3}\times\frac{n}{2}\left[2+(n-1)_4\right]=435\sqrt{3}$$

$\Rightarrow 2n+4n^2-4n=870$

$\Rightarrow 4n^2-2n-870=0$

$\Rightarrow\ 2n^2 - n - 435 = 0$

$$n = \frac{1 \pm \sqrt{1+4\times 2\times 435}}{4} = \frac{1\pm 59}{4}$$

$$\therefore\ n = \frac{1+59}{4} = 15;\ \text{or}\ n = \frac{1-59}{4} = 14.5$$

120. (d) $\left(\frac{8}{5}\right)^2 + \left(\frac{12}{5}\right)^2 + \left(\frac{16}{5}\right)^2 + \left(\frac{20}{5}\right)^2 ... + \left(\frac{44}{5}\right)^2$

$$S = \frac{16}{25}\left(2^2 + 3^2 + 4^2 + ... + 11^2\right)$$

$$= \frac{16}{25}\left(\frac{11(11+1)(22+1)}{6} - 1\right) \quad = \frac{16}{25}\times 505 = \frac{16}{5}\times 101$$

$$\Rightarrow \frac{16}{5}m = \frac{16}{5}\times 101$$

$\Rightarrow m = 101.$

121. (a) $S = (1+x)^{2016} + x(1+x)^{2015} + x^2(1+x)^{2014} + ... + x^{2015}(1+x) + x^{2016}$ ...(i)

$$\left(\frac{x}{1+x}\right) S = x(1+x)^{2015} + x^2(1+x)^{2014} + ... + x^{2016} + \frac{x^{2017}}{1+x} \quad ...(ii)$$

Subtracting (i) from (ii)

$$\frac{S}{1+x} = (1+x)^{2016} - \frac{x^{2017}}{1+x}$$

$$\therefore\ S = (1+x)^{2017} - x^{2017}$$

$$a_{17} = \text{coefficient of } x^{17} = {}^{2017}C_{17} = \frac{2017!}{17!2000!}$$

122. (d) n^{th} term of series $= \dfrac{\left[\frac{n(n+1)}{2}\right]^2}{n^2} = \frac{1}{4}(n+1)^2$

$$\text{Sum of n term} = \Sigma\frac{1}{4}(n+1)^2 = \frac{1}{4}\left[\Sigma n^2 + 2\Sigma n + n\right]$$

$$= \frac{1}{4}\left[\frac{n(n+1)(2n+1)}{6} + \frac{2n(n+1)}{2} + n\right]$$

Sum of 9 terms

$$= \frac{1}{4}\left[\frac{9\times 10\times 19}{6} + \frac{18\times 10}{2} + 9\right] = \frac{384}{4} = 96$$

123. (c) General term of given expression can be written as

$$T_r = \frac{1}{3}\left[\frac{1}{n(n+1)(n+2)} - \frac{1}{(n+1)(n+2)(n+3)}\right]$$

on taking summation both the side, we get

$$\sum_{r=1}^{5} T_r = \frac{1}{3}\left[\frac{1}{6} - \frac{1}{6.7.8}\right] = \frac{k}{3}$$

$$\Rightarrow \frac{1}{3}\times\frac{1}{6}\left(1 - \frac{1}{56}\right) = \frac{k}{3} \Rightarrow \frac{1}{3}\times\frac{1}{6}\times\frac{55}{56} = \frac{k}{3}$$

$$\Rightarrow k = \frac{55}{336}$$

124. (d) $\sum_{r=16}^{20}\left(r^2 - r - 6\right) = 7780$

125. (a) Given that $10^9 + 2.(11)(10)^8 + 3(11)^2(10)^7 + ... + 10(11)^9 = k(10)^9$

Let $x = 10^9 + 2.(11)(10)^8 + 3(11)^2(10)^7 + ... + 10(11)^9$ ...(i)

Multiplied by $\frac{11}{10}$ on both the sides

$$\frac{11}{10}x = 11.10^8 + 2.(11)^2.(10)^7 + ... + 9(11)^9 + 11^{10} \quad ...(ii)$$

Subtract (ii) from (i), we get

$$x\left(1 - \frac{11}{10}\right) = 10^9 + 11(10)^8 + 11^2\times(10)^7 + ... + 11^9 - 11^{10}$$

$$\Rightarrow\ -\frac{x}{10} = 10^9\left[\frac{\left(\frac{11}{10}\right)^{10} - 1}{\frac{11}{10} - 1}\right] - 11^{10}$$

$$\Rightarrow\ -\frac{x}{10} = (11^{10} - 10^{10}) - 11^{10} = -10^{10}$$

$\Rightarrow\ x = 10^{11} = k.10^9$ Given

$\Rightarrow\ k = 100$

126. (b) Let a, d and $2n$ be the first term, common difference and total number of terms of an A.P. respectively i.e. $a + (a+d) + (a+2d) + ... + (a + (2n-1)d)$

No. of even terms $= n$, No. of odd terms $= n$

Sum of odd terms :

$$S_o = \frac{n}{2}[2a + (n-1)(2d)] = 24$$

$\Rightarrow n\,[a + (n-1)d] = 24$ ...(i)

Sum of even terms :

$$S_e = \frac{n}{2}[2(a+d) + (n-1)2d] = 30$$

$\Rightarrow n\,[a + d + (n-1)\,d] = 30$ ...(ii)

Subtracting equation (i) from (ii), we get

$nd = 6$ ...(iii)

Also, given that last term exceeds the first term by $\frac{21}{2}$

$$a + (2n-1)\,d = a + \frac{21}{2}$$

$2nd - d = \frac{21}{2}$

$\Rightarrow 2 \times 6 - \frac{21}{2} = d \qquad (\because nd = 6)$

$d = \frac{3}{2}$

Putting value of d in equation (3)

$n = \frac{6 \times 2}{3} = 4$

Total no. of terms = $2n = 2 \times 4 = 8$

127. (a) n^{th} term of given series is

$$\frac{2n+1}{\frac{n(n+1)(2n+1)}{6}} = \frac{6}{n(n+1)}$$

Let n^{th} term, $a_n = 6\left[\frac{1}{n} - \frac{1}{n+1}\right]$

Sum of 20 terms, $S_{20} = a_1 + a_2 + a_3 + ... + a_{20}$

$$S_{20} = 6\left(\frac{1}{1} - \frac{1}{2}\right) + 6\left(\frac{1}{2} - \frac{1}{3}\right) + 6\left(\frac{1}{3} - \frac{1}{4}\right) + ...$$

$$+6\left(\frac{1}{18} - \frac{1}{19}\right) + 6\left(\frac{1}{19} - \frac{1}{20}\right) + 6\left(\frac{1}{20} - \frac{1}{21}\right)$$

$$S_{20} = \left[\left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + ... + \left(\frac{1}{18} - \frac{1}{19}\right) + \left(\frac{1}{19} - \frac{1}{20}\right) + \left(\frac{1}{20} - \frac{1}{21}\right)\right]$$

$$S_{20} = 6\left(1 - \frac{1}{21}\right) = \frac{120}{21} \qquad .....(i)$$

Given that $S_{20} = \frac{k}{21}$ .....(ii)

On comparing (i) and (ii), we get

$k = 120$

128. (c) Let $S = \frac{7}{10} + \frac{77}{100} + \frac{777}{10^3} + ... +$ up to 20 terms

$= 7\left[\frac{1}{10} + \frac{11}{100} + \frac{111}{10^3} + ... + \text{up to 20 terms}\right]$

Multiply and divide by 9

$= \frac{7}{9}\left[\frac{9}{10} + \frac{99}{100} + \frac{999}{1000} + ... \text{up to 20 terms}\right]$

$= \frac{7}{9}\left[\left(1 - \frac{1}{10}\right) + \left(1 - \frac{1}{10^2}\right) + \left(1 - \frac{1}{10^3}\right) + ... \text{up to 20 terms}\right]$

$$= \frac{7}{9}\left[20 - \frac{\frac{1}{10}\left(1 - \left(\frac{1}{10}\right)^{20}\right)}{1 - \frac{1}{10}}\right]$$

$$= \frac{7}{9}\left[\frac{179}{9} + \frac{1}{9}\left(\frac{1}{10}\right)^{20}\right] = \frac{7}{81}[179 + (10)^{-20}]$$

129. (a) Consider $1^2 + 3^2 + 5^2 + ... + 25^2$

n^{th} term $T_n = (2n-1)^2, n = 1, ... 13$

Now, $S_n = \sum_{n=1}^{13} T_n = \sum_{n=1}^{13} (2n-1)^2$

$= \sum_{n=1}^{13} 4n^2 + \sum_{n=1}^{13} 1 - \sum_{n=1}^{13} 4n = 4\sum n^2 + 13 - 4\sum n$

$= 4\left[\frac{n(n+1)(2n+1)}{6}\right] + 13 - 4\frac{n(n+1)}{2}$

Put $n = 13$, we get

$S_n = 26 \times 14 \times 9 + 13 - 26 \times 14$

$= 3276 + 13 - 364 = 2925.$

130. (c) $2^2 + 2(4)^2 + 3(6)^2 + ...$ upto 10 terms

$= 2^2 [1^3 + 2^3 + 3^3 + ...$ upto 10 terms]

$= 4 . \left(\frac{10 \times 11}{2}\right)^2 = 12100$

131. (c) Given sum is

$\frac{3}{1^2} + \frac{5}{1^2 + 2^2} + \frac{7}{1^2 + 2^2 + 3^2} + ...$

nth term = T_n

$$= \frac{2n+1}{\frac{n(n+1)(2n+1)}{6}} = \frac{6}{n(n+1)}$$

or $T_n = 6\left[\frac{1}{n} - \frac{1}{n+1}\right]$

$\therefore \quad S_n = \sum T_n = 6\sum \frac{1}{n} - 6\sum \frac{1}{n+1} = \frac{6n}{n} - \frac{6}{n+1}$

$= 6 - \frac{6}{n+1} = \frac{6n}{n+1}$

So, sum upto 11 terms means

$S_{11} = \frac{6 \times 11}{11+1} = \frac{66}{12} = \frac{33}{6} = \frac{11}{2}$

132. **(c)** $T_r = \frac{1}{1+2+3+...+r} = \frac{2}{r(r+1)}$

$$S_{10} = 2\sum_{r=1}^{10} \frac{1}{r(r+1)} = 2\sum_{r=1}^{10}\left[\frac{r+1}{r(r+1)} - \frac{r}{r(r+1)}\right]$$

$$= 2\sum_{r=1}^{10}\left(\frac{1}{r} - \frac{1}{r+1}\right)$$

$$= 2\left[\left(\frac{1}{1}-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+...+\left(\frac{1}{10}-\frac{1}{11}\right)\right]$$

$$= 2\left[1-\frac{1}{11}\right] = 2\times\frac{10}{11} = \frac{20}{11}$$

133. **(b)** nth term of the given series

$$= T_n = (n-1)^2 + (n-1)n + n^2$$

$$= \frac{((n-1)^3 - n^3)}{(n-1)-n} = n^3 - (n-1)^3$$

$$\Rightarrow S_n = \sum_{k=1}^{n}\left[k^3 - (k-1)^3\right] \Rightarrow 8000 = n^3$$

$\Rightarrow n = 20$ which is a natural number.

Hence, both the given statements are true.

and statement 2 is correct explanation for statement 1.

134. **(c)** If n is odd, the required sum is

$1^2 + 2.2^2 + 3^2 + 2.4^2 + ... + 2(n-1)^2 + n^2$

$$= \frac{(n-1)(n-1+1)^2}{2} + n^2 \quad (\because n-1 \text{ is even})$$

$$= \left(\frac{n-1}{2}+1\right)n^2 = \frac{n^2(n+1)}{2}$$

135. **(c)** Given series is $1+\frac{4}{3}+\frac{10}{9}+\frac{28}{27}+....$ n terms

$$= 1+\left(1+\frac{1}{3}\right)+\left(1+\frac{1}{9}\right)+\left(1+\frac{1}{27}\right)+.... \text{ } n \text{ terms}$$

$= (1 + 1 + 1 + ... + n$ terms$)$

$$+\left(\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+....n \text{ terms}\right)$$

$$= n + \frac{\frac{1}{3}\left(1-\frac{1}{3^n}\right)}{1-\frac{1}{3}} = n + \frac{1}{3}\times\frac{3}{2}[1-3^{-n}]$$

$$= n + \frac{1}{2}[1-3^{-n}] = n + \frac{1}{2} - \frac{1}{2.3^n}$$

136. **(c)** Given series is $\frac{1}{1+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+.....$

$$n^{\text{th}} \text{ term} = \frac{1}{\sqrt{n}+\sqrt{n+1}}$$

$$\therefore 15^{\text{th}} \text{ term} = \frac{1}{\sqrt{15}+\sqrt{16}}$$

Thus, given series upto 15 terms is

$$\frac{1}{1+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+......+\frac{1}{\sqrt{15}+\sqrt{16}}$$

This can be re-written as

$$\frac{1-\sqrt{2}}{-1}+\frac{\sqrt{2}-\sqrt{3}}{-1}+\frac{\sqrt{3}-\sqrt{4}}{-1}+......+\frac{\sqrt{15}-\sqrt{16}}{-1}$$

(By rationalization)

$$= -1+\sqrt{2}-\sqrt{2}+\sqrt{3}-\sqrt{3}+\sqrt{4}+.....-\sqrt{14}+\sqrt{15} -\sqrt{15}+\sqrt{16}$$

$$= -1+\sqrt{16} = -1+4 = 3$$

Hence, the required sum = 3

137. **(a)** The sum of the given series $1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2$ $+ 2.6^2 + ... + 2(2m)^2$ is $\frac{2m(2m+1)^2}{2} = m(2m+1)^2$

138. **(a)** Let $S = 1+\frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}+\frac{14}{3^4}+.......\infty$ ....(i)

Multiplying both sides by $\frac{1}{3}$, we get

$$\frac{1}{3}S = \frac{1}{3}+\frac{2}{3^2}+\frac{6}{3^3}+\frac{10}{3^4}+........\infty \quad ....(i)$$

Subtracting eqn. (ii) from eqn. (i), we get

$$\frac{2}{3}S = 1+\frac{1}{3}+\frac{4}{3^2}+\frac{4}{3^3}+\frac{4}{3^4}+........\infty$$

$$\Rightarrow \frac{2}{3}S = \frac{4}{3}+\frac{4}{3^2}+\frac{4}{3^3}+\frac{4}{3^4}+........\infty$$

$$\Rightarrow \frac{2}{3}S = \frac{\frac{4}{3}}{1-\frac{1}{3}} = \frac{4}{3}\times\frac{3}{2} \Rightarrow S = 3$$

139. **(d)** We know that $e^x = 1 + x + \frac{x^2}{2!}+\frac{x^3}{3!}+........\infty$

Put $x = -1$

$$\therefore e^{-1} = 1-1+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}........\infty$$

$$\therefore e^{-1} = \frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\frac{1}{5!}........\infty$$

140. **(d)** We know that

$$\frac{e^x+e^{-x}}{2} = 1+\frac{x^2}{2!}+\frac{x^4}{4!}+\frac{x^6}{6!}............$$

Putting $x = \frac{1}{2}$, we get

$$1+\frac{1}{4.2!}+\frac{1}{16.4!}+\frac{1}{64.6!}+......\infty=\frac{e^{\frac{1}{2}}+e^{\frac{-1}{2}}}{2}$$

$$=\frac{\sqrt{e}+\frac{1}{\sqrt{e}}}{2}=\frac{e+1}{2\sqrt{e}}$$

141. (b) We know that

$$e^x=1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}.....\infty$$

$$\therefore\ e=1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+.......$$

and $e^{-1}=1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+.......$

$$\therefore e+e^{-1}=2\left[1+\frac{1}{2!}+\frac{1}{4!}+....\right]$$

$$\therefore \frac{1}{2!}+\frac{1}{4!}+\frac{1}{6!}+......=\frac{e+e^{-1}}{2}-1$$

$$=\frac{e^2+1-2e}{2e}=\frac{(e-1)^2}{2e}$$

142. (b) If n is odd, the required sum is

$$1^2+2.2^2+3^2+2.4^2+......+2.(n-1)^2+n^2$$

$$=\frac{(n-1)(n-1+1)^2}{2}+n^2$$

[$\because$ $(n–1)$ is even
$\therefore$ using given formula for the sum of $(n–1)$ terms.]

$$=\left(\frac{n-1}{2}+1\right)n^2=\frac{n^2(n+1)}{2}$$

143. (d) $S_n=\frac{1}{^nC_0}+\frac{1}{^nC_1}+\frac{1}{^nC_2}+....+\frac{1}{^nC_n}$

$$t_n=\frac{0}{^nC_0}+\frac{1}{^nC_1}+\frac{2}{^nC_2}+....+\frac{n}{^nC_n}\quad ...(i)$$

$$t_n=\frac{n}{^nC_n}+\frac{n-1}{^nC_{n-1}}+\frac{n-2}{^nC_{n-2}}+....+\frac{0}{^nC_0}\quad ...(ii)$$

Adding (i) and (ii), we get,

$$2t_n=(n)\left[\frac{1}{^nC_0}+\frac{1}{^nC_1}+....\frac{1}{^nC_n}\right]=nS_n$$

$$\because\ ^nC_r={}^nC_{n-r}$$

$$\therefore \frac{t_n}{S_n}=\frac{n}{2}$$

144. (a) Let $S=\frac{1}{1.2}-\frac{1}{2.3}+\frac{1}{3.4}..............\infty$

$$T_n=\frac{1}{n(n+1)}=\left(\frac{1}{n}-\frac{1}{n+1}\right)$$

$$\therefore\ S=\left(\frac{1}{1}-\frac{1}{2}\right)-\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)-\left(\frac{1}{4}-\frac{1}{5}\right).....$$

$$=1-2\left[\frac{1}{2}-\frac{1}{3}+\frac{1}{4}-\frac{1}{5}...............\infty\right]$$

$$\left[\because \log(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+.......\infty\right]$$

$$=1-2[-\log(1+1)+1]=2\log 2-1=\log\left(\frac{4}{e}\right).$$

145. (a) $1^3-2^3+3^3-4^3+........+9^3$
$=1^3+2^3+3^3+.......+9^3-2(2^3+4^3+6^3+8^3)$

$$\left[\because \Sigma n^3=\left(\frac{n(n+1)}{2}\right)^2\right]$$

$$=\left[\frac{9\times10}{2}\right]^2-2.2^3\left[1^3+2^3+3^3+4^3\right]$$

$$=(45)^2-16.\left[\frac{4\times5}{2}\right]^2=2025-1600=425$$

146. (b) Let $P=2^{1/4}.2^{2/8}.2^{3/16}.........\infty$

$$=2^{1/4+2/8+3/16+.........\infty}$$

Now, let $S=\frac{1}{4}+\frac{2}{8}+\frac{3}{16}+........\infty$ ......(i)

$\frac{1}{2}S=\frac{1}{8}+\frac{2}{16}+........\infty$ ......(ii)

Subtracting (ii) from (i)

$$\Rightarrow\ \frac{1}{2}S=\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+........\infty$$

or $\frac{1}{2}S=\frac{a}{1-r}=\frac{1/4}{1-1/2}=\frac{1}{2}\Rightarrow S=1$

$$\therefore\ P=2^S=2$$